Master thesis for the Master of Economic Theory and Econometrics degree

Estimating the New Keynesian Phillips Curve in an Open Economy DSGE Framework

Leif Andreas Alendal

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Department of Economics

University of Oslo

Preface

This thesis was written during an internship at Norges Bank's Research Department. I wish to thank Norges Bank for inspiring working conditions. Special thanks go to my supervisors at the bank, Ida Wolden Bache and Leif Brubakk. Thanks also to Kari Elise Glenne and Kjersti Næss for proofreading. The usual disclaimer applies: All errors and inconsistencies are my own responsibility.

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1 Introduction and summary

In the last fifty years since Phillips (1958) first pointed to a possible relationship between unemployment and price and wage inflation, the Phillips curve has become one of the most intensely debated topics in macroeconomics. The recent interest in this relationship stems partly from the fact that more and more countries have adopted inflation targeting as their monetary policy regime. Understanding the evolvement of prices can also give valuable insight into the real economy, because, as Woodford (2003, p. 5) says:

> "...instability of the general level of prices *is* a good indicator of inefficiency in the real allocation of resources...because a general tendency of prices to move in the same direction...is both a cause and a symptom of systematic imbalances in resource allocation."

In resent research in open economy macroeconomics, New Keynesian dynamic stochastic general equilibrium (DSGE) models have become increasingly popular. In fact this school has been given its own name, New Open Economy Macroeconomics (NOEM).[1] The New Keynesian Phillips curve is a key equation in these models, representing the supply side of the economy. The main feature of the New Keynesian Phillips curve is that it includes expected future inflation.[2] Because of rigidities in price adjustment, firms will base their current pricing decisions on what they expect about the future.

There have been two main approaches to estimating the New Keynesian Phillips curve in the literature. One approach is single equation methods where one estimates the curve as an isolated relationship. Another approach is to estimate the curve as part of a fully specified model.

Results from single equation methods include Galí and Gertler (1999) and Galí, Gertler and López-Salido (2001) who claim that a hybrid New Keynesian Phillips curve, including both expected future inflation and lagged inflation, explains well the inflationary process in the US and the EU. They estimate different versions of the curve by General Method of Moments (GMM) and find that the purely forward looking version is rejected. The backward looking term is significant, although not very important. By contrast, Fuhrer (1997), finds that expected future inflation is unimportant in explaining price inflation in the US.

Smets and Wouters (2003) use Bayesian Maximum Likelihood to estimate the New Keynesian Phillips curve as part of a fully specified DSGE model. They use data from the Euro

[1] Good introductions to this literature are Lane (2001) and Sarno (2001).

[2] See, for example, Galí (2008) chapter 3; Walsh (2003), chapter 5 and 11; or Woodford (2003).

area and find that expected future inflation is dominant, but also that lagged inflation plays a part. Adolfson et al. (2007) use the same method as Smets and Wouters (2003), but on an open economy DSGE model. They too use data for the Euro area, and their results coincide with the ones in Smets and Wouters (2003), expected future inflation seem to be dominant.

When it comes to Norwegian data, Bårdsen et al. (2005) use a single equation approach and estimate the New Keynesian Phillips curve by GMM, and their conclusion is that the forward looking specification of the curve is rejected. Boug et al. (2006) test the New Keynesian Phillips curve with a cointegrated Vector Autoregression (VAR) model, and their results coincide with the ones in Bårdsen et al. (2005). Nymoen and Tveter (2007) estimate the version of the Phillips curve found in Norges Bank's model 1A (Husebø et al., 2004). They estimate it by GMM, and they find little evidence for the curve to be a good model for inflation dynamics in Norway. Tveter (2005) estimates domestic inflation by GMM. He estimates both a purely forward looking curve and a hybrid curve as single equations, and he identifies problems of both identification and mis-specification.

In this thesis I will estimate different versions of the New Keynesian Phillips curve as a part of a standard small open economy DSGE model. The estimation method I use is Bayesian Maximum Likelihood, and the data are Norwegian quarterly data for the period 1989Q1–2007Q4. One advantage of estimating the model as a system, is that one takes into account the cross-restrictions between the equations of the model, as opposed to single equation methods which focus on one relationship at the time. The system method therefore forces the expectations in the model to be formed in a model consistent way. Of course, this is an advantage only as long as the model is not mis-specified. The Bayesian approach also allows us to take advantage of prior information from other empirical studies, as well as from theory, in a formal way.

The supply side of the model will be represented by two types of firms, importers and producers. I assume that the law of one price is violated in the short-run. This implies that exchange rate movements will not immediately be passed through to consumer prices of imported goods. In the baseline specification I will follow Rotemberg (1982) and Hunt and Rebucci (2005) and assume quadratic price adjustment costs. In addition, I will consider an alternative specification following Galí and Gertler (1999). They assume that only a fraction of producers get to change their price each period[3] and that some of them follow a rule of thumb in their price setting. The demand side will consist of a continuum of equal consumers who maximize discounted expected utility, where utility in each period depends

[3] This assumption was first introduced by Calvo (1983).

on consumption and leisure. The consumers are assumed to have habit persistence in their consumption preferences. The government collects lump-sum taxes and spends them on domestic goods, and the central bank is assumed to follow a simple Taylor rule in interest rate setting. The rest of the world will be regarded as one big economy, and it will be approximated by autoregressive processes.[4]

The benchmark DSGE model includes flexible hybrid Phillips curves based on Rotemberg pricing behavior. I will compare this specification to alternative specifications of the New Keynesian Phillips curve, including a purely forward looking version. To compare model fit I use the posterior odds ratio.

My main findings are that expected future inflation is dominant in the New Keynesian Phillips curve. This result applies to both domestic and imported inflation. When comparing the models, the more flexible the Phillips curves are towards putting weight on expected future inflation, the better the model fits the data. A model with a hybrid New Keynesian Phillips curve with a restriction of fifty-fifty on the coefficients on expected future inflation and lagged inflation gives the poorest data fit. A classic purely forward looking New Keynesian Phillips curve gives better data fit than a flexible hybrid curve. This, however, may be a result of the fact that the purely forward looking curve contains fewer estimated parameters than the hybrid, flexible curve and that it has better priors by construction. I also estimate two models with slightly more ad hoc versions of the price-setting rules. One version is a homogeneous[5] hybrid Phillips curve in which the coefficients on both expected future inflation and lagged inflation are allowed to vary between zero and one. The other is similar, but where the homogeneity restriction is relaxed. The results are the same as for the benchmark model, the expected future inflation term is dominant. For the non-homogeneous model, the sum of the coefficient estimates on the inflation terms in the domestic price curve is not that far away from unity, but more so for the import price curve. However, the relative data fit between these two models indicates that homogeneity is not a too strong assumption.

The structure of the thesis is as follows: Section 2 elaborates on the origin of the Phillips curve and the development towards the New Keynesian version. Then, I derive two different versions of the New Keynesian Phillips curve, one based on the Rotemberg assumption of quadratic price adjustment costs and one based on the Calvo assumption of random opportunity for price adjustment. Finally, Section 2 presents a selection of empirical results from other studies. Section 3 derives the rest of the model. In Section 4 I explain the estimation method and describe the data set used in the estimation. The results are presented in Section

[4] AR(1)-processes.

[5] That is, that the coefficients on the lead and lag term sum to one (a vertical long run Phillips curve).

5, and Section 6 concludes.

I use Matlab and Dynare[6] for data transformation and estimation.

2 The Phillips Curve

In this section I will look at the historical background and development of the Phillips curve. I will then derive two different versions of the New Keynesian Phillips curve, based on two different assumptions about price setting behavior. I take a look at different methods that have been used to estimate New Keynesian Phillips curves in the literature, and, finally, I give a brief overview of the main results.

2.1 Historical background

In 1958 *Economica* printed an article by Alban William Phillips with the title *The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957* (Phillips, 1958). By analyzing the British economy, Phillips had found an inverse relationship between the unemployment rate and wage growth.[7] In a diagram of wage growth and unemployment, he fitted a convex curve showing that when unemployment was low, wage growth was high and vice versa. His conclusion was that it seemed as though keeping demand at a level which allowed wages to grow with productivity[8] – and thereby keeping product prices stable – the resulting unemployment rate would be just above 2 per cent. If one tried to keep demand at a level that gave constant wages, the resulting unemployment rate would be about 5 per cent. Thus, there seemed to be a trade-off between wage growth and unemployment which could be exploited by governments. Phillips ended his article with the following two sentences:

> "These conclusions are of course tentative. There is need for much more detailed research into the relations between unemployment, wage rates, prices and productivity."

The trade-off relationship was soon accepted by many researchers, and it was believed that by accepting higher price inflation, one could achieve lower unemployment. The curve

[6]See Dynare homepage `http://www.cepremap.cnrs.fr/dynare/` or Griffoli (2007).

[7]With the exception of war times, in which import prices rose rapidly and initiated wage-price spirals. Phillips therefore ignored years with rapid import price increases in his analysis.

[8]Assumed by Phillips to be 2 per cent annually.

that Phillips had constructed between wage rate growth and unemployment was named the Phillips curve. It was also expressed as a relationship between price inflation and unemployment.[9]

In the 1970s, several countries experienced high inflation and high unemployment at the same time – a situation that seemingly contradicted the Phillips curve. Milton Friedman (1968) argued that Phillips should have looked at real, and not nominal wages, as it is the real income for employees that matters. If prices were to increase more than anticipated as a result of, for example, expansionary monetary policy, real wages would be lower than expected. Then, even though employment would increase in the short run as a result of increased demand for labor, workers would update their expectations and demand higher wages in the future, resulting in lowered demand for labor. Thus, to maintain the increase in employment, monetary policy would have to be even more expansionary in the future, that is, the inflation rate would have to *accelerate*. The trade-off between unemployment and prices was not between unemployment and a *high* inflation rate, but a *rising* inflation rate.

Friedman and Edmund S. Phelps (1967) argued that there existed a level of unemployment at which there would be neither upward nor downward pressure on real wages as a result of expectation formation. The theory of the *non-accelerating inflation rate of unemployment* (NAIRU) was born.[10] Monetary policy could only alter the unemployment rate by surprise inflation and the effect would only be temporary. Then, in 1976 Robert E. Lucas Jr. wrote his famous article *Econometric policy evaluation: A critique* (Lucas, 1976), where he argued that historical relationships between two (or more) economic variables would break down if the conditions for economic decisions changed. Phillips curves estimated on historical data would be useless to predict the future evolution in unemployment and prices/wages if, for example, monetary or fiscal policy changed, as economic agents then would adjust their behavior to the new policy. Lucas emphasized the need to model expectations explicitly and to formulate models in terms of structural, or *deep*, parameters, characterizing underlying preferences and technology.

Finn E. Kydland and Edward C. Prescott initiated a new era in macroeconomic modeling with their seminal article *Time to Build and Aggregate Fluctuations* in 1982 (Kydland and Prescott, 1982). Since then, micro founded macro models, where agents make optimal choices based on their preferences and constraints and on rational expectations about the future,

[9] Irving Fisher had in fact discovered this relationship already in the 1920s, but still the curve was named after Phillips. See Fisher (1973).

[10] Friedman called it the *natural* rate of unemployment, but he emphasized that he did not think that it was unchangeable, but influenced by for example minimum wages and the strength of unions.

have become very important in two schools of macroeconomics, namely Real Business Cycle Theory (RBC) and New Keynesian Economics. Both RBC models and New Keynesian models are dynamic, stochastic, general equilibrium models. The main difference between RBC and New Keynesian models is that, in contrast to RBC theory, the New Keynesians believe that there exist rigidities in nominal wages and prices, so that in the short-run, monetary policy has real effects and employment levels can be socially sub-optimal. Thus government intervention in demand can help achieve a more favorable production level in the short run.

In this thesis I will focus on the New Keynesian perspective[11] and derive a simple DSGE model for a small open economy with nominal rigidities. One of the key equations in this model is the New Keynesian Phillips curve representing the supply side of the economy. The main difference between the New Keynesian Phillips curve and the original Phillips curve is that the New Keynesian Phillips curve is forward looking: current inflation depends on the expectation of future inflation. Another difference is that in the New Keynesian Phillips curve, the driving variable in the inflation process is real marginal costs,[12] not unemployment.

2.2 The New Keynesian Phillips curve

The key assumption underlying the New Keynesian Phillips curve is that it is either costly, or in some way difficult, to adjust prices every period. This could be due to some kind of *menu costs* of changing prices. When for example Ikea distributes a new catalog, it is plausible that it takes into account expectations of future costs when the prices in the catalog are set, since it would be costly to distribute a new catalog every time input prices changed.

There have been several suggestions on how to model price rigidity. Taylor (1979, 1980) assumed that contracts are made for several periods at the time. Then, if only a fraction of prices and wages are changed every period, both the past and the expected future will play a role in optimal price and wage setting. Calvo (1983) assumed that firms are not able to change their prices every period, and that the probability that a firm is able to change its prices in a given period, is determined by an exogenous Poisson process. In this case the duration of prices will be random, and firms need to form expectations about the future to

[11]For more on RBC theory, see for example Kydland and Prescott (1990), Rebelo (2001) or King and Rebelo (2000).

[12]It is also common to use the output gap (the difference between actual and potential output). The link between the output gap and unemployment was first proposed by Okun (1962), see also Prachowny (1993). See Galí and Gertler (1999) and Galí et al. (2001) for discussions of which driving variables to use when estimating the New Keynesian Phillips curve.

set optimal prices. Rotemberg (1982) assumes quadratic costs of changing prices. In this case it may not be optimal to change prices to what is optimal seen from the current period only, because next period's optimal price might be different, and then the cost of changing the prices could exceed the gain. Therefore, one has to form expectations of future optimal prices when setting prices today.[13] Here, I will first focus on Rotemberg's assumption and assume that there exist costs of changing prices relative to both steady state inflation and previous period's aggregate inflation. This will give hybrid versions of the New Keynesian Phillips curves, where not only expectation of future prices, but also previous period's prices play a role in price settings. Following Galí and Gertler (1999), I will also discuss a Calvo representation of the New Keynesian Phillips curve which assumes that some firms set prices according to a backward looking rule of thumb.

When we want to look at the economy of a small, open country, we need to distinguish between domestic and imported inflation. Several empirical studies have rejected the law of one price, at least in the short-run (see for example Campa and Goldberg, 2005 and Goldberg and Knetter, 1997). In line with Smets and Wouters (2002) I assume that there is complete pass-through to import prices at the docks, but that the importers face adjustment costs in their own price setting, so that there will be incomplete pass-through to consumer prices of imported goods.

2.2.1 Deriving the New Keynesian Phillips curve assuming quadratic costs of price adjustment

The domestic economy has two types of firms, domestic producers and importers, and a continuum of each type, indexed from zero to one. The domestic producers sell their products to domestic and foreign consumers while importers only sell their products in the domestic market.

The consumption index is given by

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

where α is related to the degree of openness of the domestic economy. $C_{H,t}$ and $C_{F,t}$ represent

[13] For more on different approaches to modeling price rigidities, see for example Walsh (2003).

aggregate consumption of domestic and foreign goods, given by

$$C_{H,t} = \left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon_{H,t}-1}{\varepsilon_{H,t}}} di \right]^{\frac{\varepsilon_{H,t}}{\varepsilon_{H,t}-1}} \text{ and } C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\varepsilon_{F,t}-1}{\varepsilon_{F,t}}} di \right]^{\frac{\varepsilon_{F,t}}{\varepsilon_{F,t}-1}},$$

where both domestic and foreign goods are defined as CES aggregates of a continuum of differentiated goods, indexed by (i). The elasticity of substitution between domestic and foreign goods is given by $\eta > 0$, and the elasticities of substitution between the different types of domestic and foreign goods are given by ε_H and ε_F,[14] respectively. Optimal demand for each category of goods are[15]

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \tag{2}$$

$$\text{and } C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t, \tag{3}$$

where

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\varepsilon_{H,t}} di \right]^{\frac{1}{\varepsilon_{H,t}-1}} \text{ and } P_{F,t} = \left[\int_0^1 P_{F,t}(i)^{1-\varepsilon_{F,t}} di \right]^{\frac{1}{\varepsilon_{F,t}-1}}$$

are the price indices of domestic and foreign goods, respectively. The aggregate price level, or the consumer price index (CPI), is

$$P_t \equiv \left[(1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{4}$$

In the same way we find optimal demand for each individual good within the two categories to be

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon_{H,t}} C_{H,t} \text{ and } C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon_{F,t}} C_{F,t}.$$

Domestic producers produce domestic goods by a constant return to scale technology defined by $Y_t(i) = Z_t^Y N_t(i)$, where labor, N_t, is the only input factor, and Z_t^Y is total factor

[14] The εs are assumed to be greater than 1 to ensure that profit maximizing monopolistic firms operate with positive price markups in steady state.

[15] See Appendix B.1 for detailed derivations.

productivity. Z_t^Y is assumed to follow the process

$$\ln\left(\frac{Z_t^Y}{Z^Y}\right) = \rho^Y \ln\left(\frac{Z_{t-1}^Y}{Z^Y}\right) + \xi_t^Y,$$

where ρ^Y $(0 \leq \rho^Y \leq 1)$ measures the degree of persistence and ξ_t^Y is an i.i.d. shock. Throughout the thesis, a variable without a time subscript denotes the steady state value of that variable.[16] Domestic goods are sold both to domestic and foreign households and also to the domestic government. We assume that the law of one price holds in the foreign economy and that foreign consumers have identical preferences for domestic goods as domestic consumers. Foreign demand for domestic goods, $C_{H,t}^f$, is then

$$C_{H,t}^f = \alpha^f \left(\frac{P_{H,t}}{S_t P_{F,t}^f}\right)^{-\eta} C_t^f, \tag{5}$$

where C_t^f is total foreign demand. Total demand for domestic goods, $C_{H,t}^T$, becomes

$$C_{H,t}^T = C_{H,t} + C_{H,t}^f + G_t = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha^f \left(\frac{P_{H,t}}{S_t P_{F,t}^f}\right)^{-\eta} C_t^f + G_t, \tag{6}$$

where the first term is domestic consumers' demand for domestic goods, the second term is foreign consumers' demand for domestic goods and the last term, G, denotes government spending.

In line with Rotemberg (1982) and Hunt and Rebucci (2005), I assume that the firms face quadratic costs of price adjustment. The costs, Γ_{PC_H}, arise both from changes in inflation relative to steady state inflation and from changes in firm i's inflation relative to previous period's aggregate inflation

$$\Gamma_{PC_H,t}(i) \equiv \frac{\phi_{C_{H1}}}{2}\left(\frac{P_{H,t}(i)}{\pi P_{H,t-1}(i)} - 1\right)^2 + \frac{\phi_{C_{H2}}}{2}\left(\frac{P_{H,t}(i)/P_{H,t-1}(i)}{P_{t-1}/P_{t-2}} - 1\right)^2. \tag{7}$$

Here $\phi_{C_{H1}}$ is a parameter measuring the costs of adjusting prices relative to steady state inflation. $\phi_{C_{H2}}$ is the parameter measuring the costs of changing the inflation rate relative to aggregate inflation in the previous period. The optimal price in period t follows from

$$\max_{P_{H,t}(i)} E_t \sum_{\tau=t}^{\infty} \left\{D_{t,\tau}\left[P_{H,\tau}(i) - MC_{H,\tau}(i)\right] C_{H,\tau}^T(i)\left[1 - \Gamma_{PC_h,\tau}(i)\right]\right\},$$

[16]Steady state is defined as a state where the next period's expected state stays constant to the current state. That is, all endogenous variables stay constant.

where $D_{t,\tau}$ is the stochastic discount factor, which will be defined later. The marginal cost for producer i, $MC_{H,t}$, is given by $W_t(i)/Z_t^Y$. The first order condition for the optimal price is[17]

$$\begin{aligned}
0 = {} & [1-\Gamma_{PC_H,t}(i)]\left[P_{H,t}(i)\,(1-\varepsilon_{H,t})+\varepsilon_{H,t}\frac{W_t(i)}{Z_t^Y}\right] \\
& -\left[P_{H,t}(i)-\frac{W_t(i)}{Z_t^Y}\right]\frac{\phi_{C_{H1}}P_{H,t}(i)}{\pi P_{H,t-1}(i)}\left(\frac{P_{H,t}(i)}{\pi P_{H,t-1}(i)}-1\right) \\
& -\left[P_{H,t}(i)-\frac{W_t(i)}{Z_t^Y}\right]\frac{\phi_{C_{H2}}P_{H,t}(i)/P_{H,t-1}(i)}{P_{t-1}/P_{t-2}}\left(\frac{P_{H,t}(i)/P_{H,t-1}(i)}{P_{t-1}/P_{t-2}}-1\right) \\
& + E_t D_{t,t+1}\frac{C^T_{H,t+1}(i)}{C^T_{H,t}(i)}\left[P_{H,t+1}(i)-\frac{W_t(i)}{Z_t^Y}\right] \\
& \times\left[\begin{array}{c}\left(\frac{\phi_{C_{H1}}P_{H,t+1}(i)}{\pi P_{H,t}(i)}\right)\left(\frac{P_{H,t+1}(i)}{\pi P_{H,t}(i)}-1\right) \\ +\left(\frac{\phi_{C_{H2}}P_{H,t+1}(i)/P_{H,t}(i)}{P_t/P_{t-1}}\right)\left(\frac{P_{H,t+1}(i)/P_{H,t}(i)}{P_t/P_{t-1}}-1\right)\end{array}\right].
\end{aligned} \tag{8}$$

We see that with no adjustment costs, that is, $\phi_{C_{H1}}=\phi_{C_{H2}}=\Gamma_{PC_H,t}(i)=0$, the only term remaining in (8) is $0=P_{H,t}(i)\,(1-\varepsilon_{H,t})+\varepsilon_{H,t}W_t(i)/Z_t^Y$ which yields the simple optimal monopoly price as a markup on marginal cost

$$P_{H,t}(i)=\frac{\varepsilon_{H,t}}{\varepsilon_{H,t}-1}MC_{H,t}(i).$$

The monopolistic competition assumption is essential in New Keynesian modeling. It ensures that firms are willing to change output levels when demand changes, even if they do not change their prices.

Importers all buy the same input at given world price $P^f_{F,t}$. Each importer then puts a unique brand on it and sells the final product in the domestic market. The importers have monopoly power in the market for their own (branded) good. Their price setting optimization problem is identical to the one for domestic producers, with the exception that marginal cost for importers is given by $S_tP^f_{F,t}$, where S_t is the nominal exchange rate. The first order

[17] Detailed derivation can be found in Appendix B.3.

condition for importers is

$$
\begin{aligned}
0 = & \left[1-\Gamma_{PC_F,t}(i)\right]\left[P_{F,t}(i)\left(1-\varepsilon_{F,t}\right)+\varepsilon_{F,t}S_tP^f_{F,t}\right] \\
& -\left[P_{F,t}(i)-S_tP^f_{F,t}\right]\frac{\phi_{C_{F1}}P_{F,t}(i)}{\pi P_{F,t-1}(i)}\left(\frac{P_{F,t}(i)}{\pi P_{F,t-1}(i)}-1\right) \\
& -\left[P_{F,t}(i)-S_tP^f_{F,t}\right]\frac{\phi_{C_{F2}}P_{F,t}(i)/P_{F,t-1}(i)}{P_{t-1}/P_{t-2}}\left(\frac{P_{F,t}(i)/P_{F,t-1}(i)}{P_{t-1}/P_{t-2}}-1\right) \\
& +E_tD_{t,t+1}\frac{C_{F,t+1}(i)}{C_{F,t}(i)}\left[P_{F,t+1}(i)-S_tP^f_{F,t}\right] \\
& \times\left[\begin{array}{c}\left(\frac{\phi_{C_{F1}}P_{F,t+1}(i)}{\pi P_{F,t}(i)}\right)\left(\frac{P_{F,t+1}(i)}{\pi P_{F,t}(i)}-1\right)+ \\ \left(\frac{\phi_{C_{F2}}P_{F,t+1}(i)/P_{F,t}(i)}{P_t/P_{t-1}}\right)\left(\frac{P_{F,t+1}(i)/P_{F,t}(i)}{P_t/P_{t-1}}-1\right)\end{array}\right].
\end{aligned} \tag{9}
$$

The law of one price holds between foreign goods and imports at the wholesale level, hence there is complete pass-through of exchange rate movements to wholesale prices. But because of the price adjustment costs in the local markets, the pass-through of exchange rate movements to consumer prices of imported goods will be incomplete in the short run. With flexible prices – that is, with no adjustment costs – we see that the optimal price will be

$$P_{F,t}(i)=\frac{\varepsilon_{F,t}}{\varepsilon_{F,t}-1}S_tP^f_{F,t},$$

and thus there will be complete pass-through of exchange rate movements all the way to consumer prices.

By log-linearizing equations (8) and (9) around the steady state, assuming that all firms within the two sectors are equal, we get the following two Phillips curves for domestic and imported inflation[18]

$$
\begin{aligned}
\widehat{\pi}^H_t = & -\frac{\varepsilon_H}{\phi_{C_{H1}}+(1+\beta)\,\phi_{C_{H2}}}\widehat{\varepsilon}_{H,t}+\frac{\varepsilon_H\left(\varepsilon_H-1\right)}{\phi_{C_{H1}}+(1+\beta)\,\phi_{C_{H2}}}\left(\widehat{w}_t-\widehat{Z}^Y_t-\widehat{p}_{H,t}\right) \\
& +\frac{\phi_{C_{H2}}}{\phi_{C_{H1}}+(1+\beta)\,\phi_{C_{H2}}}\widehat{\pi}^H_{t-1}+\beta\frac{\phi_{C_{H1}}+\phi_{C_{H2}}}{\phi_{C_{H1}}+(1+\beta)\,\phi_{C_{H2}}}E_t\widehat{\pi}^H_{t+1}
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
\widehat{\pi}^F_t = & -\frac{\varepsilon_F}{\phi_{C_{F1}}+(1+\beta)\,\phi_{C_{F2}}}\widehat{\varepsilon}_{F,t}+\frac{\varepsilon_F\left(\varepsilon_F-1\right)}{\phi_{C_{F1}}+(1+\beta)\,\phi_{C_{F2}}}\left(\widehat{Q}_t-\widehat{p}_{F,t}\right) \\
& +\frac{\phi_{C_{F2}}}{\phi_{C_{F1}}+(1+\beta)\,\phi_{C_{F2}}}\widehat{\pi}^F_{t-1}+\beta\frac{\phi_{C_{F1}}+\phi_{C_{F2}}}{\phi_{C_{F1}}+(1+\beta)\,\phi_{C_{F2}}}E_t\widehat{\pi}^F_{t+1}
\end{aligned} \tag{11}
$$

[18]See Appendix C.6 for detailed derivations.

All variables with a "$\widehat{\ }$" are percentage deviations from the steady state level of the corresponding variable. Small characters are real variables (that is, divided by the price index (4), e.g. $p_H = P_H/P$). The inflation rates in the two prices are defined as $\pi_t^i = P_t^i/P_{t-1}^i$, and Q is the real exchange rate, defined as $Q = SP_F^f/P$. β is the discount factor.

We see that inflation depends negatively on movements in the elasticity of demand between different types of goods, ε. An increase in the elasticity means less market power for the firms and thus a lower mark-up. I therefore refer to $\widehat{\varepsilon}$ as a shock to market power. We also see that if real marginal costs increase, the firm will increase its price. Depending on β and the ϕs, the coefficients on expected future inflation and lagged inflation can vary between zero and one. For a given discount factor, β, close to unity, the coefficient on the lead term can vary between one half and β, and the coefficient on lagged inflation must be between zero and one half. If there are no costs of adjusting inflation relative to steady state inflation, that is the ϕ_1s are zero, then the coefficients on lagged inflation and expected future inflation reduce to $1/(1+\beta)$ and $\beta/(1+\beta)$, respectively. This means that for β close to unity, both coefficients will be approximately one half. This version of the price change costs is used by Norges Bank in their Norwegian Economy Model (NEMO) (Brubakk et al., 2006). By introducing costs of deviating from steady state inflation, we see that we get a more flexible Phillips curve. By setting the ϕ_2s to zero, corresponding no costs of changing prices relative to past inflation, we get the purely forward looking New Keynesian Phillips curve.

2.2.2 Calvo pricing

Galí and Gertler (1999) introduce backward looking rule of thumb behavior in a Calvo pricing framework. They assume, in a traditional Calvo manner, that only a fraction $1-\theta$ of the firms will be able to adjust their prices in the current period. Of these, however, only a fraction $1-\omega$ will behave in traditional Calvo way and optimize their price with respect to expected future marginal costs when given the opportunity to change prices. A fraction ω will set prices following a simple rule based on the previous period's reset price. The aggregate price level will follow

$$p_t = \theta p_{t-1} + (1-\theta)\,\overline{p}_t^*, \tag{12}$$

where $\overline{p}_t^*$ is an index for prices that have been changed in the current period. This index can be written as

$$\overline{p}_t^* = (1-\omega)\,p_t^f + \omega p_t^b, \tag{13}$$

where p^f and p^b represent prices chosen by the optimizing firms and the backward looking firms, respectively. The optimal price for forward looking firm (i) is[19]

$$p^f_{i,t} = (1 - \theta\beta) \sum_{j=0}^{\infty} \theta^i \beta^i E_t \widehat{mc}^n_{t+j}, \tag{14}$$

where $\widehat{mc}^n_t$ is nominal marginal costs in percentage deviations from steady state. For the backward looking firms, the price is set according to

$$p^b_t = \overline{p}^*_{t-1} + \pi_{t-1}. \tag{15}$$

The price setting in the current period is only based on information available in date $t-1$ or earlier. The rule implies that the price will not deviate from the optimal price in the steady state. By combining equations (12)–(15), we get the following Phillips curve

$$\widehat{\pi}_t = \lambda \widehat{mc}_t + \gamma_f E_t \widehat{\pi}_{t+1} + \gamma_b \widehat{\pi}_{t-1,} \tag{16}$$

where

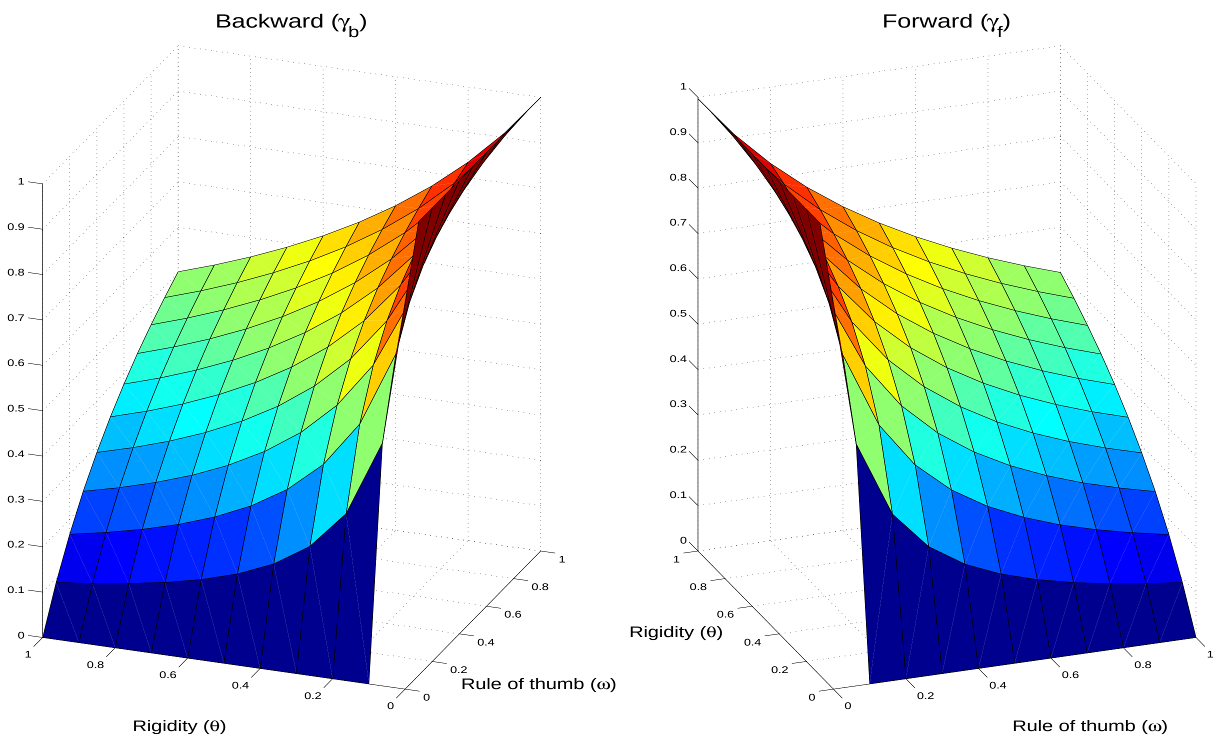
$$\begin{aligned}
\lambda &\equiv \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\theta + \omega\left[1 - \theta(1-\beta)\right]} \\
\gamma_f &\equiv \frac{\beta\theta}{\theta + \omega\left[1 - \theta(1-\beta)\right]} \\
\gamma_b &\equiv \frac{\omega}{\theta + \omega\left[1 - \theta(1-\beta)\right]}.
\end{aligned}$$

Galí and Gertler (1999) point out that if no firms set prices by the backward looking rule (that is, ω is zero) we will be left with the original, purely forward looking, New Keynesian Phillips curve. But we can also see that for a given share of rule of thumb firms, ω, and discount factor, β, the weight on lagged inflation in the New Keynesian Phillips curve is *decreasing* in the degree of price stickiness (that is, increasing in θ). This can be seen from Figure 1 which shows the coefficients on the inflation terms in (16) (γ_f and γ_b) for different levels of price rigidity, θ, and different shares of rule-of-thumb behaving firms, ω, when $\beta = 0.993$. In addition we see that if the fraction of price setters that use a backward looking rule increases, then the coefficient on the lagged term will increase. If we add some rule of thumb firms, the Calvo pricing assumption implies a more flexible New Keynesian Phillips curve than the

[19] See Appendix B.4 for detailed derivation.

quadratic adjustment cost assumption, in that it allows the lead term coefficient to be less than one half and the lag term coefficient to be larger than one half.

Figure 1: Phillips curve coefficients. Calvo pricing with rule of thumb behavior



$\beta = 0.993$

2.3 Empirical studies

In the fifty years since Phillips wrote his article, different versions of the Phillips curve have been estimated by different methods and on different data sets. We can distinguish between two main approaches, single equation methods and system methods. Single equation methods have been the most popular. Recently, however, partly due to the development of computers, system estimation methods have become increasingly popular – in particular Bayesian Maximum Likelihood. The latter will be described in detail in Section 4.

The main difference between system estimation and single equation methods is that with a system approach, we estimate the complete model, not just certain equilibrium equations one at the time. We can then take advantage of restrictions that exist between other equations and the one we investigate. On the other hand, this can also be a disadvantage if our model is mis-specified.

2.3.1 Single equation estimation

A popular method in the empirical literature is GMM. The GMM estimator minimizes the distance between the theoretical moments of the model and the corresponding moments in the sample (see, for example, Canova, 2007, chapter 5). To ensure identification of all parameters, one needs at least one instrument for every endogenous variable. But even if this criterion is met, GMM can suffer from *weak identification* if the instruments are only weakly correlated with the regressors. Then the regression results can be misleading even if the sample size is considered to be satisfactory. In addition, there could be problems of mis-specification in the sense that added instruments may be highly correlated with the endogenous regressor, as they should be, but without being exogenous – leading to spurious identification (Mavroeidis, 2005).

Galí and Gertler (1999) estimate equation (16) with different restrictions on US quarterly data for the period 1960Q1–1997Q4 by GMM. They find evidence that the forward looking term in the Phillips curve is very important for price development,[20] and that the lag term is significant, but not important.[21] They also find evidence that marginal costs are significant in price setting behavior and that prices seem to be rather rigid. Mavroeidis (2005) has criticized these results, arguing that under the assumption of the model being correctly specified, the parameters are not identified (or only weakly so). Fuhrer (1997) finds little role for the lead term in the Phillips curve for the US when a lag term is added, but concludes that a hybrid version could be reasonable for policy simulation.

Batini et al. (2005) use UK data for the period 1972Q3–1999Q2 to estimate both a purely forward looking New Keynesian Phillips curve and a hybrid version, with and without homogeneity restrictions, in an open economy model. They too use GMM, and they get an estimate of 0.69 on the lead term coefficient in the purely forward looking version of the model. When estimating the unrestricted version of the hybrid curve, the coefficient estimate for the lead term becomes 0.48 and the lag term coefficient is 0.15. The restricted version is rejected by an F-test.

On Norwegian data, Bårdsen et al. (2005) estimate a purely forward looking New Keynesian Phillips curve by GMM for the period 1972Q2–2001Q1. They find no evidence for

[20] They estimate that about 60–80 per cent of firms set prices in a forward looking manner.
[21] Similar results are found for the Euro Area in Galí et al. (2001).

the New Keynesian Phillips curve. Tveter (2005) estimates a purely forward looking and a hybrid curve for domestic inflation on quarterly Norwegian data for the period 1979Q3–2003Q3. The result is an insignificant coefficient on the wage share (which is used as a proxy for real marginal costs) and autocorrelation in the residuals. He concludes that there exist problems of both identification and mis-specification.

Bache and Naug (2007) estimate a variety of import Phillips curves on UK and Norwegian data by GMM, and they find little evidence of forward looking behavior in the UK data, but more so in the Norwegian data. For both countries they find little evidence of indexation in price setting.

To sum up, the results from the empirical literature using single equation methods span from expected inflation being important to not playing a role at all in the New Keynesian Phillips curve.

2.3.2 System estimation

There is an increasing literature estimating the New Keynesian Phillips curve and New Keynesian import price equations as parts of fully specified DSGE models. A common estimation method in this literature is Bayesian Maximum Likelihood.

Smets and Wouters (2003) estimate a full DSGE model by Bayesian Maximum Likelihood on data from the Euro area. Their results point towards considerable rigidities in both prices and wages. They find the forward looking component in the Phillips curve to be dominant, but also that inflation depends on lagged inflation.

Lindé (2005) argues that single equation methods, like GMM, most likely will produce biased estimates, and that a system approach should be used. He estimates a New Keynesian Phillips curve by Full Information Maximum Likelihood on US data for the period 1960Q1–1997Q4. The conclusion is that there is a clear role for both forward and backward looking behavior in the inflation process.

Adolfson et al. (2007) estimate an open economy DSGE model on Euro data, using Bayesian estimation. They assume Calvo price setting in both the domestic sector and the import sector, but with an indexation rule depending on previous period's inflation and the inflation target. This gives a hybrid New Keynesian Phillips curve with the same restrictions on the coefficients on the forward and backward terms as the Rotemberg pricing assumption we derived above. The coefficient on the lead term is free to vary between one half and one, and the coefficient for lagged inflation can vary between zero and one half. They find evidence of price rigidities both in the domestic and import goods sectors, and it looks like the domestic prices are considerably more rigid than import prices. The coefficient on the lead term in the Phillips curve is estimated to be a little over 0.8 for both domestic and imported inflation, and thus a little less than 0.2 on the lag term. This is in accordance with Galí and Gertler (1999) and Galí et al. (2001).

Boug et al. (2006) test different versions of the New Keynesian Phillips curve on quarterly Norwegian data for the period 1983Q1–2001Q1. Both single equation and system approaches are used, including cointegrated VAR models. Their conclusion is that a forward looking term is superfluous in inflation modeling, and that other empirical results should be re-evaluated by use of cointegration tests.

3 The complete model

In this section I will first derive the demand side of the model. This is represented by both domestic and foreign households and a domestic government. I then specify an interest rate rule for the central bank and define the equilibrium of the economy. Finally, I will briefly explain how the model is solved.

3.1 Households

Households consist of a continuum of infinitely-lived individuals, indexed by j, who consume aggregates of domestic (C_H) and imported (C_F) goods. The consumers are assumed to maximize the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_t^j - hC_{t-1}\right)^{1-\sigma}}{1-\sigma} - \frac{\left(N_{t+i}^j\right)^{1+\varphi}}{1+\varphi} \right], \tag{17}$$

where β is the discount factor, C_t^j is household j's consumption, C_{t-1} is previous period's aggregate consumption and h $(0 < h < 1)$ measures the importance of habits. N_t is labor input, and the parameters σ and φ represent the inverse of elasticity of intertemporal substitution and the Frisch elasticity of labor supply, respectively. The elasticity of intertemporal substitution measures the consumer's willingness to shift consumption between periods. When this elasticity is low, the consumers are said to be risk averse. Thus, σ is also a measure of relative risk aversion.[22] The Frisch elasticity of labor supply measures the response in

[22] Relative risk aversion is often measured by $-\frac{cu''}{u'}$, which yields σ with a utility function like (17).

hours of a wage change when marginal utility of consumption is kept fixed. Thus it measures the substitution effect of a wage change. Habit formation is introduced to capture inertia in consumers' response to changing conditions in the economy. The result is slower adjustment in consumption and output, and this gives the desired *hump shape* form of consumption and output in responses to shocks (see for example Fuhrer, 2000).

Utility maximization by household j is done subject to the following budget constraint

$$C_t^j + \frac{B_t^j}{(1+r_t)\,P_t} + \frac{S_t B_t^{f,j}}{\left(1+r_t^f\right)\Phi(A_t)P_t} = \frac{B_{t-1}^j}{P_t} + \frac{S_t B_{t-1}^{f,j}}{P_t} + \frac{W_t}{P_t}N_t^j + X_t^j - T_t^j, \tag{18}$$

where B_t^j and $B_t^{f,j}$ are one period bond holdings in domestic and foreign currency respectively, and r_t and r_t^f are domestic and foreign short term nominal interest rates. T is the lump sum tax. To ensure stationary bond holdings I follow Benigno (2001) and add a risk premium on foreign bonds.[23] The risk premium is represented by the function $\Phi(A_t) = e^{-\phi A_t + Z_t^B}$ which is strictly decreasing in the domestic economy's aggregate real holdings of foreign bonds defined as $A_t \equiv S_t B_t^f / P_t$. To account for uncertainty in the risk premium I add the shock variable Z^B that follows the process

$$Z_t^B = \rho^B Z_{t-1}^B + \xi_t^B,$$

where ρ^B measures the degree of persistence and ξ_t^B is an i.i.d. shock. Even though the premium depends on bond holdings, the households treat it as given when they optimize because their individual influence is negligible. Real profits in the economy is divided equally among all households, this yields the lump sum term X_t^j in the budget constraint. To solve the household's optimization problem, we form the lagrangian

$$\mathcal{L} = E_t \sum_{i=0}^{\infty} \beta^i \left\{ \begin{array}{c} \left[\frac{\left(C_{t+i}^j - hC_{t+i-1}\right)^{1-\sigma}}{1-\sigma} - \frac{\left(N_{t+i}^j\right)^{1+\varphi}}{1+\varphi} \right] \\ -\lambda_{1,t+i} \left[\begin{array}{c} C_{t+i}^j + \frac{B_{t+i}^j}{(1+r_{t+i})P_{t+i}} + \frac{S_{t+i}B_{t+i}^{f,j}}{\left(1+r_{t+i}^f\right)\Phi(A_{t+i})P_{t+i}} \\ -\frac{B_{t+i-1}^j}{P_{t+i}} - \frac{S_{t+i}B_{t+i-1}^{f,j}}{P_{t+i}} - \frac{W_{t+i}}{P_{t+i}}N_{t+i}^j \end{array} \right] \end{array} \right\},$$

where λ is the Lagrange multiplier. By maximizing with respect to C_{t+i}^j, B_{t+i}^j, $B_{t+i}^{f,j}$ and N_{t+i}^j, combining first order conditions and rearranging, we get the following optimality conditions:

[23] This can be ensured by different methods. For an overview see Scmitt-Grohé and Uribe (2003).

The Euler equation

$$\beta(1+r_t)E_t\left[\left(\frac{C^j_{t+1}-hC_t}{C^j_t-hC_{t-1}}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right]=1, \tag{19}$$

says that the optimal plan for consumption is such that marginal utility of consumption today is equal to the discounted expected marginal utility tomorrow.

The optimal quantity of foreign bonds will be determined from expected depreciation or appreciation of the domestic currency, the risk premium and the difference in gross interest rates between the two economies. This is represented in the uncovered interest rate parity (UIP) condition

$$\frac{1+r_t}{1+r^f_t}=E_t\left[\frac{S_{t+1}}{S_t}\right]\Phi\left(A_t\right). \tag{20}$$

The intratemporal optimality condition

$$\frac{W_t}{P_t}=\frac{N^\varphi_t}{\left(C^j_t-hC_{t-1}\right)^{-\sigma}}, \tag{21}$$

says that the optimal amount of labor supply is determined by the real wage and the marginal rate of substitution between leisure and consumption. If one chooses to work one hour less, one gets more utility from the extra hour of leisure. But one must also renounce some consumption as a result of the reduction in income.

The stochastic discount factor is defined as

$$D_{t,\tau}\equiv E_{\tau-1}\beta\left[\left(\frac{C^j_\tau-hC_{\tau-1}}{C^j_{\tau-1}-hC_{\tau-2}}\right)^{-\sigma}\frac{P_{\tau-1}}{P_\tau}\right],$$

and we see that in steady state, it is equal to β.

3.2 Equilibrium

Households receive all profits from domestic firms and importers. The households also receive all revenues from price adjustment costs. Foreigners are assumed not to hold domestic bonds, so when aggregating the budget constraint (18), net supply of domestic bonds are zero. The aggregate budget constraint then reads

$$C_t+\frac{S_tB^f_t}{\left(1+r^f_t\right)\Phi(A_t)P_t}=\frac{S_tB^f_{t-1}}{P_t}+\frac{W_t}{P_t}N_t+X_t-T_t\ . \tag{22}$$

Substituting in for the production function, real profits

$$X_t = \left(\frac{P_{H,t}}{P_t} - \frac{W_t}{P_t Z_t^Y}\right)\left(C_{H,t} + C_{H,t}^f\right) + \left(\frac{P_{F,t}}{P_t} - \frac{S_t P_{F,t}^f}{P_t}\right) C_{F,t} \tag{23}$$

and the market clearing condition in the market for domestic goods

$$Y_t = C_{H,t} + C_{H,t}^f + G_t, \tag{24}$$

I obtain[24]

$$\frac{S_t B_t^f}{\left(1 + r_t^f\right)\Phi(A_t)} - S_t B_{t-1}^f = P_{H,t} C_{H,t}^f - S_t P_{F,t}^f C_{F,t}. \tag{25}$$

The change in net foreign bond holdings is equal to net profits in foreign trade. Or, in other words, if the domestic country runs a current account surplus, the surplus will be put in foreign bonds.

3.3 The government

Government spending, G, is only spent on domestic goods. It is financed with a lump sum tax T and assumed to evolve according to

$$\ln\left(\frac{G_t}{G}\right) = \rho^G \ln\left(\frac{G_{t-1}}{G}\right) + \xi_t^G. \tag{26}$$

The central bank is assumed to follow a simple Taylor rule for interest rate setting

$$\widehat{R}_t = \omega_r \widehat{R}_{t-1} + \left(\frac{1-\omega_r}{R}\right)\left[\omega_\pi \widehat{\pi}_t + \omega_y\left(\widehat{y}_t - \widehat{y}_{t-1}\right)\right] + \xi_t^r, \tag{27}$$

where R is the gross interest rate defined as $R = 1 + r$, ω_r is the degree of interest rate smoothing, ω_π is the weight on current inflation, ω_y is the weight on output growth and ξ^r is an i.i.d. shock.

[24] See Appendix B.5 for detailed derivation.

3.4 Estimated model

By log-linearizing[25] equations (1)–(3), (19)–(21), (24)–(26) and the production function, and using (10), (11) and (27), we have the following approximated model which will be used for estimation

$$\widehat{C}_{H,t} = \widehat{C}_t - \eta \widehat{p}_{H,t} \tag{28}$$

$$\widehat{C}_{F,t} = \widehat{C}_t - \eta \widehat{p}_{F,t} \tag{29}$$

$$\widehat{C}^f_{H,t} = \widehat{C}^f_t - \eta \left(\widehat{p}_{H,t} - \widehat{Q}_t \right) \tag{30}$$

$$\widehat{Y}_t = \widehat{Z}^Y_t + \widehat{N}_t \tag{31}$$

$$\widehat{Y}_t = \frac{C_H}{Y} \widehat{C}_{H,t} + \frac{C^f_H}{Y} \widehat{C}^f_{H,t} + \frac{G}{Y} \widehat{G}_t \tag{32}$$

$$\widehat{C}_t = \frac{h}{(1+h)} \widehat{C}_{t-1} + \frac{1}{(1+h)} E_t \widehat{C}_{t+1} - \frac{(1-h)}{(1+h)} \frac{1}{\sigma} \left(\widehat{r}_t - E_t \widehat{\pi}_{t+1} \right) \tag{33}$$

$$\widehat{C}_t = (1 - \gamma_c) \widehat{C}_{H,t} + \gamma_c \widehat{C}_{F,t} \tag{34}$$

$$\widehat{R}_t - \widehat{R}^f_t = E_t \widehat{Q}_{t+1} - \widehat{Q}_t + E_t \widehat{\pi}_{t+1} - E_t \widehat{\pi}^f_{t+1} - \phi A_t + Z^B_t \tag{35}$$

$$\frac{Q \widehat{b}^f_t}{R^f} - Q \widehat{b}^f_{t-1} = p_H C^f_H \left(\widehat{p}_{H,t} + \widehat{C}^f_{H,t} \right) - Q C_F \left(\widehat{Q}_t + \widehat{C}_{F,t} \right) \tag{36}$$

$$\frac{1}{\varphi} \widehat{w}_t - \frac{\sigma}{\varphi(1-h)} \widehat{C}_t + \frac{\sigma h}{\varphi(1-h)} \widehat{C}_{t-1} = \widehat{N}_t \tag{37}$$

$$\begin{aligned} \widehat{\pi}^H_t = & - \frac{\varepsilon_H}{\phi_{C_{H1}} + (1+\beta) \phi_{C_{H2}}} \widehat{\varepsilon}_{H,t} + \frac{\varepsilon_H (\varepsilon_H - 1)}{\phi_{C_{H1}} + (1+\beta) \phi_{C_{H2}}} \left(\widehat{w}_t - \widehat{Z}^Y_t - \widehat{p}_{H,t} \right) \\ & + \frac{\phi_{C_{H2}}}{\phi_{C_{H1}} + (1+\beta) \phi_{C_{H2}}} \widehat{\pi}^H_{t-1} + \beta \frac{\phi_{C_{H1}} + \phi_{C_{H2}}}{\phi_{C_{H1}} + (1+\beta) \phi_{C_{H2}}} E_t \widehat{\pi}^H_{t+1} \end{aligned} \tag{38}$$

$$\begin{aligned} \widehat{\pi}^F_t = & - \frac{\varepsilon_F}{\phi_{C_{F1}} + (1+\beta) \phi_{C_{F2}}} \widehat{\varepsilon}_{F,t} + \frac{\varepsilon_F (\varepsilon_F - 1)}{\phi_{C_{F1}} + (1+\beta) \phi_{C_{F2}}} \left(\widehat{Q}_t - \widehat{p}_{F,t} \right) \\ & + \frac{\phi_{C_{F2}}}{\phi_{C_{F1}} + (1+\beta) \phi_{C_{F2}}} \widehat{\pi}^F_{t-1} + \beta \frac{\phi_{C_{F1}} + \phi_{C_{F2}}}{\phi_{C_{F1}} + (1+\beta) \phi_{C_{F2}}} E_t \widehat{\pi}^F_{t+1} \end{aligned} \tag{39}$$

[25] The equations are linearized by a first order Taylor approximation around the steady state. A first order Taylor approximation of $f(x_t, y_t)$ around its steady state $f(x, y)$ is $f(x_t, y_t) \approx f(x, y) + f_x(x, y)(x_t - x) + f_y(x, y)(y_t - y)$. See Appendix C for detailed derivations.

$$\widehat{R}_t = \omega_r \widehat{R}_{t-1} + \left(\frac{1-\omega_r}{R}\right)[\omega_\pi \widehat{\pi}_t + \omega_y(\widehat{y}_t - \widehat{y}_{t-1})] + \xi_t^r. \tag{40}$$

γ_c is import's share of consumption. The variables $\widehat{\varepsilon}_H$, $\widehat{\varepsilon}_F$, $\widehat{C}^f$, $\widehat{R}^f$, $\widehat{\pi}^f$ and $\widehat{G}$ are assumed to follow AR(1)-processes.

3.5 Solving the model

The following five steps are involved when solving and analyzing nonlinear dynamic stochastic models (see Uhlig, 1999)

1. Derive the model's equilibrium conditions.
2. Find steady state of the model.
3. Log-linearize the equilibrium conditions around the steady state.
4. Solve for the recursive equilibrium law of motion – that is, find optimal policy rules.
5. Analyze the solution.

The equilibrium conditions were derived above. The log-linear approximations around the steady state are given by (28)–(40) plus the AR(1)-processes.

There exist several ways to solve linear rational expectations models, see for example Dejong and Dave (2007), Blanchard and Kahn (1980) or Uhlig (1999). Anderson (2006) compares several solution techniques and finds that as long as the Blanchard-Kahn conditions (which will be described below) are satisfied, the techniques will give equivalent solutions. I will use the built-in routines in Dynare to solve the model.

In log-linearized form, the DSGE model constitutes a set of first order conditions and constraints which can be represented by

$$AE_t y_{t+1} + By_t + Cy_{t-1} + Du_t = 0, \tag{41}$$

where y is a vector of the state variables, both endogenous and exogenous, u is a vector of shocks, and A–D are matrices capturing the coefficients.

Now, according to proposition 1–3 in Blanchard and Kahn (1980), if this system has more eigenvalues outside the unit circle than there are non-predetermined variables, then there exists no stable solution to the system. If the number of eigenvalues outside the circle are less than the numbers of non-predetermined variables, there will be an infinity of solutions

– and if the numbers coincide, there will be a unique stable solution. The solution will consist of a set of optimal policy rules for the endogenous variables, which can be written in the form

$$y_t = Fy_{t-1} + Gu_t.$$

We can represent the solution in state-space form, with a set of equations for the optimal policy rules and a set of measurement equations

$$y_t = Fy_{t-1} + Gu_t \tag{42}$$

$$y_t^{obs} = Hy + Hy_t, \tag{43}$$

where y_t^{obs} is the vector of observable variables, $E_t[u_t u_t'] = \Sigma_u$ and y is a vector for the steady state values of the non-observables. Note that I have not included any measurement errors in the measurement equations. This state space representation will be used to find the log-likelihood function in the estimation procedure described in Section 4.

4 Estimation

In this section I will elaborate on Bayesian Maximum Likelihood, specify the priors for the estimation and examine the data set.

Based on Monte Carlo simulations, Lindé (2005) argues and shows "...that single equations methods, e.g. GMM, are likely to produce imprecise and biased estimates" of the coefficients in the New Keynesian Phillips curve, and that a system approach is more sensible. Also, as described in Section 2.3.1, estimating the New Keynesian Phillips curve by GMM can suffer from weak identification.[26] I will therefore try to estimate the New Keynesian Phillips curve as part of a model, using Bayesian Maximum Likelihood.

Additional advantages of Bayesian Maximum Likelihood are that the posterior distributions reflect uncertainty about the parameters. One can thus, for example, answer questions regarding the probability of a parameter being in some region. Bayesian Maximum Likelihood also allows the researcher to incorporate prior information about the parameters in a formal way. These are some of the reasons why Bayesian methods have become increasingly popular in macro modeling. For more thorough introductions to Bayesian analysis, see for example Lancaster (2004), Canova (2007) chapter 9 and 11, Hamilton (1994) chapter 12, or

[26] I have run several GMM estimations on the Phillips curves I have derived in Section 2 on the data set described later in this section. The results are very sensitive to the choice of instrument sets.

An and Schorfheide (2007) for a detailed discussion of Bayesian estimation of DSGE models.

4.1 Estimation method

The key building blocks of Bayesian estimation are the priors, the likelihood density function and Bayes' theorem. The basic principle of Bayesian estimation is to combine prior information with information from the data as represented by the likelihood function. As a result of empirical studies and economic theory, we may have prior beliefs about the parameters of a model. In Bayesian estimation, we put our beliefs into prior densities, and then we confront our beliefs with data. The result is a posterior density which is obtained by Bayes' theorem and which is a function of our priors and the likelihood density produced by the data. By choosing the distribution of the priors, we can decide how much weight should be put on our beliefs. The more certain we are of a parameter, the tighter the prior we choose, and then less weight is put on the data.

The data gives the likelihood density

$$\mathcal{L}\left(\theta_{\mathcal{M}}|Y_T,\mathcal{M}\right) \equiv p\left(Y_T|\theta_{\mathcal{M}},\mathcal{M}\right) = \frac{p\left(\theta_{\mathcal{M}};Y_T\right)}{p\left(\theta_{\mathcal{M}}\right)} \Leftrightarrow p\left(\theta_{\mathcal{M}};Y_T\right) = p\left(Y_T|\theta_{\mathcal{M}},\mathcal{M}\right)p\left(\theta_{\mathcal{M}}\right),$$

where $\mathcal{M}$ is a specific model, Y_T is observed data until time T, and $\theta_{\mathcal{M}}$ is a vector of parameters for model $\mathcal{M}$. The likelihood for the parameter set $\theta_{\mathcal{M}}$ is the probability of observing the data set Y_T given the parameters $\theta_{\mathcal{M}}$ in model $\mathcal{M}$. What we want to find is how probable the parameters $\theta_{\mathcal{M}}$ are, given the data Y_T. Combining the likelihood with the prior density $p\left(\theta\right)$, we use Bayes' theorem to find the posterior density

$$p(\theta_{\mathcal{M}}|Y_T,\mathcal{M}) = \frac{p\left(\theta_{\mathcal{M}};Y_T\right)}{p\left(Y_T|\mathcal{M}\right)} = \frac{p\left(Y_T|\theta_{\mathcal{M}},\mathcal{M}\right)p\left(\theta_{\mathcal{M}}|\mathcal{M}\right)}{p\left(Y_T|\mathcal{M}\right)}.$$

Since the marginal density of the data conditional on the model, $p\left(Y_T|\mathcal{M}\right)$, is constant, the posterior kernel is proportionate to the posterior density

$$p(\theta_{\mathcal{M}}|Y_T,\mathcal{M}) \propto p\left(Y_T|\theta_{\mathcal{M}},\mathcal{M}\right)p\left(\theta_{\mathcal{M}}|\mathcal{M}\right) \equiv \mathcal{K}(\theta_{\mathcal{M}}|Y_T,\mathcal{M}).$$

To illustrate this, we can look at a simple example presented by Stéphane Adjemian

(2005, p. 7–10).[27] We have a data generating process

$$y_t = \mu + \varepsilon_t, \;\; t = 1, ..., T$$

where ε is a white noise process, i.e. $\varepsilon_t \sim \mathcal{N}(0, 1)$. Then the likelihood is given by

$$p\left(Y_T|\mu\right) = (2\pi)^{-\frac{T}{2}} \exp\left(-\frac{1}{2}\sum_{t=1}^{T}\left(y_t - \mu\right)^2\right).$$

The maximum likelihood estimator for μ is

$$\widehat{\mu}_{ML,T} = \frac{1}{T}\sum_{t=1}^{T} y_t \equiv \overline{y}$$

Since the variance of $y_t = 1$, the variance of $\widehat{\mu}_{ML,T}$ is simply $\frac{1}{T}$. If we choose a normally distributed prior with expectation μ_0 and variance σ_μ^2, we will have the following posterior kernel:

$$p\left(\mu|Y_T\right) \propto \left(2\pi\sigma_\mu^2\right)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\frac{\left(\mu - \mu_0\right)^2}{\sigma_\mu^2}\right) \times (2\pi)^{-\frac{T}{2}} \exp\left(-\frac{1}{2}\sum_{t=1}^{T}\left(y_t - \mu\right)^2\right),$$

which is equivalent to

$$p\left(\mu|Y_T\right) \propto \exp\left(-\frac{\left(\mu - E\left[\mu\right]\right)^2}{V\left[\mu\right]}\right).$$

So the posterior distribution is normal and has variance and expectation:

$$V\left[\mu\right] = \frac{1}{\left(\frac{1}{T}\right)^{-1} + \sigma_\mu^{-2}}$$
$$E\left[\mu\right] = \frac{\left(\frac{1}{T}\right)^{-1}\widehat{\mu}_{ML,T} + \sigma_\mu^{-2}\mu_0}{\left(\frac{1}{T}\right)^{-1} + \sigma_\mu^{-2}}.$$

We see that if we have no prior beliefs, i.e. $\sigma_\mu^2 \to \infty$, the expectation converges to the maximum likelihood estimate $\widehat{\mu}_{ML,T}$, with variance $\frac{1}{T}$. If we are certain in our beliefs, and do not want to put any weight on what the data gives, i.e. $\sigma_\mu^2 \to 0$, the expectation converges to μ_0.

[27] The same example is also used in the Dynare User Guide by Tommaso Mancini Griffoli, see http://www.cepremap.cnrs.fr/juillard/mambo/download/manual/Dynare_UserGuide_WebBeta.pdf.

For a larger model that is nonlinear in the parameters, the computation of the posterior density becomes infinitely more complex, and we have to simulate the posterior by, for instance, a Markov chain. I will return to this below.

A great advantage of the linear approximation of the model, is that one can use the Kalman filter to analyze the state-space representation of the policy functions. The linearized policy functions are still non-linear in the parameters, but since they are linear in the variables, the Kalman filter can be used to estimate the likelihood function, which we need in order to find the posterior kernel. The Kalman filter works recursively and estimates the state of our system when some of the state variables are unobservable. Detailed descriptions of the filter are given in Hamilton (1994), chapter 13, and in Canova (2007), chapter 6.

Following Griffoli (2007), recursions based on the state space representation in equations (42) and (43) can be written

$$
\begin{aligned}
\zeta_t &= y_t^{obs} - \overline{y}^{obs} - H y_t \\
S_t &= H P_t H' \\
K_t &= F P_t F' S_t^{-1} \\
y_{t+1} &= F y_t + K_t \zeta_t \\
P_{t+1} &= F P_t (F - K_t H)' + G \Sigma_u G',
\end{aligned}
$$

for $t = 1, ..., T$, with initial conditions y_1 and P_1, where P is the error covariance matrix of the state estimate. The filter first predicts the state variables for time t based on information available in time $t-1$, where the residual is ζ. The residual covariance matrix is S. Then the filter calculates the optimal Kalman gain K, and updates the estimate of the state y and the error covariance matrix P.

This gives the log-likelihood

$$
\log \mathcal{L}\left(\theta | Y_T^{obs}\right) = \frac{Tk}{2} \log 2\pi - \frac{1}{2} \sum_{t=1}^{T} \log |S_t| - \frac{1}{2} \sum_{t=1}^{T} \zeta_t' S_t^{-1} \zeta_t,
$$

where θ consists of the k parameters we want to estimate (k =sum of elements in $\theta_{\mathcal{M}}$ and vech(Σ_u)[28])

After we have specified our priors, we have an estimate for the posterior kernel as a

[28] *vech* is a vectorization of a symmetric matrix, excluding the upper portion.

function of the likelihood and the prior densities. In log terms, it can be written as

$$\ln \mathcal{K}\left(\theta|Y_T^{obs}\right) = \ln \mathcal{L}\left(\theta|Y_T^{obs}\right) + \ln p\left(\theta\right),$$

where Y_T^{obs} is the set of observable endogenous variables.

Since the posterior distribution is nonlinear in the parameters and thus too complicated to calculate analytically, it has to be simulated. For this I use a Metropolis-Hastings algorithm. The Metropolis Hastings algorithm is a Markov chain Monte Carlo simulation algorithm which can be used to simulate any distribution. This can be done as long as, for a given value, we are able to calculate a function proportional to the density at that value. This is exactly what the estimated kernel enables us to do. The algorithm consists of four steps, in which the first is an initial step, and the next three are repeated a chosen number of times to ensure convergence.

1. Choose an initial vector for parameters θ^0, for example the calculated mode.
2. Draw a random θ^* from the jumping distribution $J\left(\theta^*|\theta^{t-1}\right) = \mathcal{N}\left(\theta^{t-1}, c\Sigma_m\right)$, where Σ_m is the inverse of the Hessian[29] from the mode computation.
3. Compute a ratio for acceptance
$$r = \frac{p\left(\theta^*|Y_T\right)}{p\left(\theta^{t-1}|Y_T\right)} = \frac{\mathcal{K}\left(\theta^*|Y_T\right)}{\mathcal{K}\left(\theta^{t-1}|Y_T\right)}.$$
4. Draw a random number α from a uniform distribution $U\left(0,1\right)$. Then, if α is smaller than r, keep θ^* as θ^t and update the jumping distribution. Thus
$$\theta^t = \begin{cases} \theta^* & \text{with probability } \min\{r,1\} \\ \theta^{t-1} & \text{otherwise} \end{cases}.$$

We see that if the calculated kernel with the proposed values θ^* is larger than the kernel with last periods values θ^{t-1}, we keep the proposed values for sure and update the expectation of the jumping distribution. On the other hand, if the kernel evaluated at θ^* is smaller than for the last period's values θ^{t-1}, we still want to keep the proposal with a certain probability. The reason for this is that there can be both local and global maxima in the distribution. A consequence of this is that if we only keep proposals that give higher kernels, we could

[29]The Hessian is a matrix of second order derivatives of a function.

end up in a local maximum. We therefore keep some proposals, even if they produce smaller kernels, because they might lead us to an even higher kernel further on in the chain.

If we choose the scale of the variance in the jumping distribution c too small, we can experience the same problem. We then risk getting stuck in a local area of the distribution, and we cannot be sure that it is not a local maximum. It could take a long time to reach convergence. Too large jumps are not wanted either, as we then will get a too small acceptance rate, and a lot of time will be wasted in areas of the distribution that are of low interest. For more about Markov chains and Metropolis-Hastings algorithms, see Gilks et.al (1996).

Once we have obtained the posterior distribution, we can use it to compare different models' predictive abilities. By integrating out the parameters from the posterior kernel, we get the marginal density of the data conditional on the model,

$$p\left(Y_T|\mathcal{M}\right) = \int_{\Theta_{\mathcal{M}}} p(\theta_{\mathcal{M}}|Y_T, \mathcal{M})d\theta_{\mathcal{M}},$$

where $\Theta_{\mathcal{M}}$ is the entire parameter space for model $\mathcal{M}$. This measure can be obtained from the Metropolis Hastings simulation, and together with any prior beliefs we might have of the different models' probabilities, we can form the posterior probability for each model i as

$$p\left(\mathcal{M}_i|Y_T\right) = \frac{p\left(Y_T|\mathcal{M}_i\right)p\left(\mathcal{M}_i\right)}{\sum_i p\left(Y_T|\mathcal{M}_i\right)p\left(\mathcal{M}_i\right)},$$

where $p\left(\mathcal{M}_i\right)$ is our prior for model i. A common way to compare two models is then to calculate the ratio of their two posterior probabilities

$$\frac{p\left(\mathcal{M}_1|Y_T\right)}{p\left(\mathcal{M}_2|Y_T\right)} = \frac{p\left(Y_T|\mathcal{M}_1\right)p\left(\mathcal{M}_1\right)}{p\left(Y_T|\mathcal{M}_2\right)p\left(\mathcal{M}_2\right)}.$$

This is called the posterior odds, where $p\left(Y_T|\mathcal{M}_1\right)/p\left(Y_T|\mathcal{M}_2\right)$ is called Bayes' factor, and $p\left(\mathcal{M}_1\right)/p\left(\mathcal{M}_2\right)$ is the prior odds. If we have equal priors for both models, we can go straight to Bayes' factor and compare the marginal density of each model to get an impression on which model predicts the data best. See, for example, An and Schorfheide (2007) or Kass and Raftery (1995).

4.2 Priors

In the estimation I will focus on the parameters entering the New Keynesian Phillips curve and the parameters in the shock processes. Several parameters in the model will be kept fixed during estimation. In other words they will be given infinitely tight priors. The reason for this is mainly that they are unlikely to be identified with the data set used in the estimation. The calibration will be based on long-run averages of the data, economic theory and estimation results from other studies. Most of the calibrated parameters will be in line with the ones set in Brubakk et al. (2006) for Norges Bank's NEMO.

The discount factor β is set to 0.993. This gives a steady state annual real interest rate of 2.85 per cent which is in accordance with estimates of the Norwegian neutral real interest rate (Bernardsen, 2005). Brubakk et al. (2006) argue that since Norway is a more specialized economy than many others that have been subject to micro studies, the elasticity of substitution between domestic and foreign goods η should be set at a relatively low value (1.1).[30] This reflects that for many goods imported to Norway, there exist few, if any, substitutes. The degree of openness α is chosen to be 0.32. This gives a steady state import share of consumption of 0.32 which corresponds fairly well to the current weight on imported goods in the consumer price index.

The Frisch elasticity of labor supply is assumed to be 0.33 which gives a value of 3 for φ. This is in accordance with both Brubakk et al. (2006) and Galí (2008, chapter 7). Real Business Cycle theory often assumes the Frisch elasticity to be one, implying a lot more flexibility in working hours – on the other hand, micro studies indicate that the elasticity should be lower. The elasticity of substitution between different types of domestic and foreign goods, ε_H and ε_F, are both assumed to be 6. This also corresponds with both Brubakk et al. (2006) and Galí (2008, chapter 7). It yields steady state price mark-ups of 1.2 which is a moderate degree of market power. It is common to assume that households have log preferences in consumption. This means that the substitution and income effect on saving from interest changes cancel out. I will follow this by assuming σ to be 1.

In accordance with the original Taylor rule, ω_π is set to 1.5 (Taylor, 1993). The weight on output growth, ω_y, is set to 0.5. In addition, the smoothing parameter ω_r is set to 0.7.

Lindé et al. (2004) estimates the log-linear UIP condition (35) and conclude that a reasonable value for the parameter for the risk premium on holding foreign bonds ϕ lies in the interval 0–0.115. I set ϕ to a relatively low 0.0002 to let the risk premium ensure stationary bond holdings in the long run.

It is common to set the habit formation parameter h to be about 0.7. This is also in line with the estimates achieved by Adolfson et al. (2005) and Boldrin et al. (2001). I set h to

[30] This parameter is usually in the range 1–5 in models for US and EU. For example, Adolfson et al. (2007) set this parameter to 5. Naug (2002) estimates it to be 1.5 for Norway. Here chosen equal to the one in Brubakk et al. (2006).

Table 1: Calibrated parameters

Parameter	Description	Value
α	Degree of openness	0.32
β	Discount factor	0.993
σ	Intertemporal elastisity of substitution	1
φ	Inverse Frisch elastisity of labour supply	3
η	Elastisity of sub. betw. domestic and foreign goods	1.1
ε_H	Elastisity of sub. betw. different types of domestic goods	6
ε_F	Elastisity of sub. betw. different types of foreign goods	6
ω_π	Weight on inflation gap in taylor rule	1.5
ω_y	Weight on output gap in taylor rule	0.5
ω_r	Degree of interest rate smoothing in taylor rule	0.7
ϕ	Parameter for risk premium on holding foreign bonds	0.0002
h	Degree of habit formation in consumption	0.75

Table 2: Priors for shocks

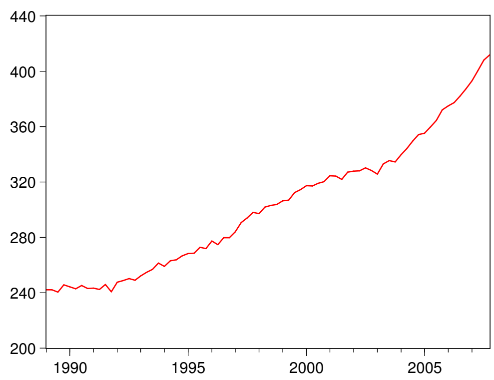
Parameter	Description	Distribution	Mean	S.D.
ρ_y	Persistence Y	Beta pdf	0.5	0.2
ρ_b	Persistence b	Beta pdf	0.5	0.2
ρ_G	Persistence G	Beta pdf	0.5	0.2
$\rho_{\varepsilon H}$	Persistence ε_H	Beta pdf	0.5	0.2
$\rho_{\varepsilon F}$	Persistence ε_F	Beta pdf	0.5	0.2
stderr ξ^Y		Inv. gam. pdf	0.02	inf.
stderr ξ^b		Inv. gam. pdf	0.01	inf.
stderr ξ^G		Inv. gam. pdf	0.012	inf.
stderr ξ^r		Inv. gam. pdf	0.0025	inf.
stderr $\xi^{\varepsilon H}$		Inv. gam. pdf	0.05	inf.
stderr $\xi^{\varepsilon F}$		Inv. gam. pdf	0.05	inf.

0.75 which corresponds to the prior set in Brubakk et al. (2006). All the calibrated priors are found in Table 1.

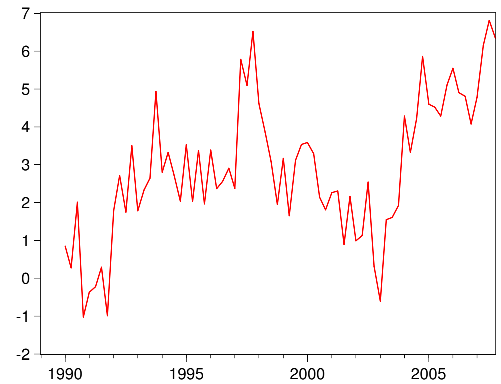
The priors for the standard errors and persistence of shocks are given in Table 2. The estimated shocks are: a productivity shock, a monetary policy shock, a shock to government spending, shocks in market power for the two types of producers and finally a shock to the risk premium on holding foreign bonds.

I will discuss the priors for the estimated Phillips curve parameters in Section 5. All prior distributions are plotted together with the posterior distributions and the modes in Appendix A.

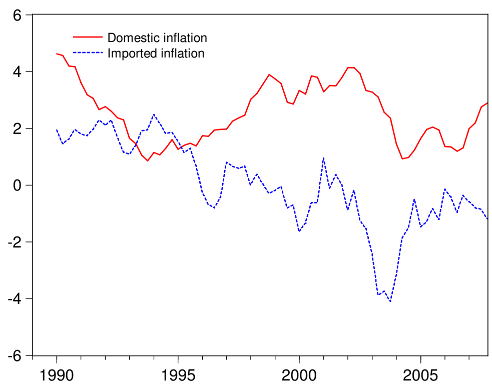
Figure 3: The data



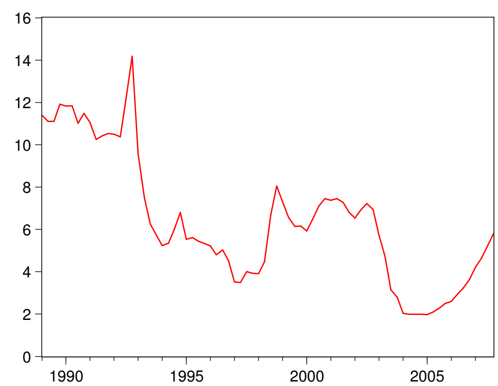
(a) GDP. Billion NOK. Fixed prices



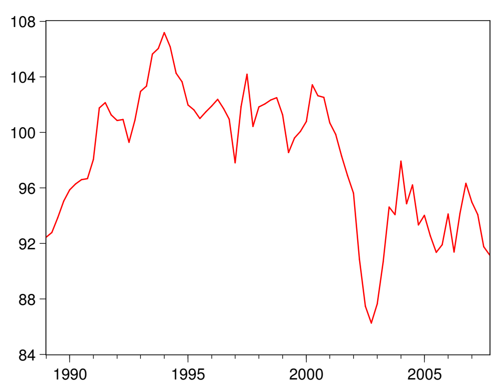
(b) GDP growth. Annual per cent



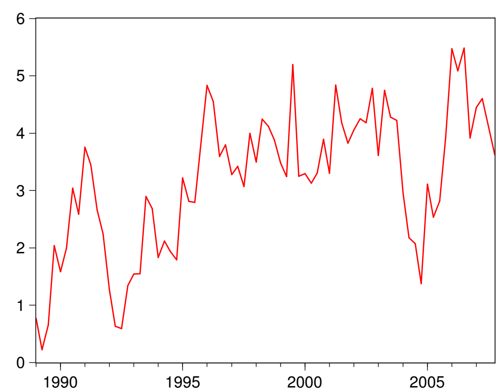
(c) Inflation. Annual per cent



(d) Interest rate. Annual per cent



(e) Real exchange rate



(f) Real wage growth. Annual per cent

4.3 The data

All series are collected from Norges Bank's database. The series are: the total consumer price index (P)(adjusted for taxes and energy prices), the consumer price index for domestic goods (P_H), the consumer price index for imported goods (P_F), gross domestic product in the mainland economy (GDP, Y in the model), the real exchange rate (Q), nominal wage income per hour (W) and short term (3 months) interest rates (r).[31] I use the nominal wages series together with total consumer price index to form a series for real wages. The price indices and the series for gross domestic product are seasonally adjusted. The real exchange rate is constructed from an import weighted nominal exchange rate based on 44 countries (I-44) together with consumer price indices for Norway and 25 trading partners.

I use data for the period 1989Q1–2007Q4 for the estimation. Choosing the estimation period is not trivial. Most of the series I use start earlier than 1989 thus there is more information available. But, given that I assume that the parameters of the model are constant over time, it could be more sensible to choose a shorter estimation period. Faced with this trade-off, I have chosen to focus on the period 1989–2007. We can see from Figure 3 that GDP has grown considerably during the period. The mean annual growth in GDP over the period is 2.88 per cent, while for the last four years the mean annual growth rate is 5 per cent. In addition we see that domestic prices have been growing relatively stable around today's target[32] of 2.5 per cent. Imported inflation, on the other hand, has been more or less steadily decreasing, with an exceptional deflation of about 4 per cent in 2003. The short-term interest rate starts out at a high level at the beginning of the period when Norway followed a fixed exchange rate regime. There is a peak in 1992 as a result of pressure on the exchange rate, followed by a shift to a lower level when Norges Bank had to let the Krone float. The real exchange rate depreciates some during the first three years of the estimation period and then is more stable for the following six or seven years. Then it appreciates quite a bit and stays at a lower level till the end of the period. The real wage is growing over the whole period, and the mean annual real wage growth for the period is 3.2 per cent.

Since the model is stationary, we need to transform data to remove the trends. By taking first differences of the two price indices, I get the gross inflation rates (π_H and π_F). To relate them to the percentage deviation from steady state, which is the variable in the estimated model, I subtract 1. The real exchange rate is used in log-form, while the interest rate is divided by 400. I use the first differences of the log of GDP and the real wage.

[31]The names of the series in the database are: QSA_PCPIJAE QSA_PCPIJAEI QSA_PCPIJAEIMP QUA_QI44 QSA_YMN QUA_RN3M WILMN_PCT_Q.

[32]Norway introduced inflation targeting in March 2001.

I then have the following vector of observables:

$$Y_t^{obs} = \{\pi_{H,t}, \pi_{F,t}, \Delta Y_t, Q_t, \Delta w_t, r_t\}.$$

All observables will also be demeaned before estimation.

5 Results

This section summarizes the estimation results. For all simulations, the Metropolis Hastings algorithm is set to pick 1.5 million draws from the jumping distribution. For robustness, I run two different chains of the algorithm with different starting values. The jump scale parameter, c, is chosen to give about the desired acceptation rate of 0.2–0.4 (Griffoli, 2007). Diagnostic plots for the estimations are reported in Appendix A. The posterior distributions are plotted against the prior densities and the modes. In addition are plots of aggregate convergence diagnostics from the Markov chains for each estimator reported. Ideally, the two chains should converge to a constant value. That would indicate that the parameter distribution has converged. Highly volatile chains indicate some kind of problem, one possible reason is that the priors are poor. When calculating the mode, I have checked that the posterior density has curvature in the region of the mode of each parameter.

5.1 Benchmark model

The benchmark model is the model based on the assumption of quadratic price adjustment costs. The Phillips curves are

$$\begin{aligned}
\widehat{\pi}_t^H = & -\frac{\varepsilon_H}{\phi_{C_{H1}} + (1+\beta)\,\phi_{C_{H2}}}\widehat{\varepsilon}_{H,t} + \frac{\varepsilon_H(\varepsilon_H - 1)}{\phi_{C_{H1}} + (1+\beta)\,\phi_{C_{H2}}}\left(\widehat{w}_t - \widehat{Z}_t - \widehat{p}_{H,t}\right) \\
& + \frac{\phi_{C_{H2}}}{\phi_{C_{H1}} + (1+\beta)\,\phi_{C_{H2}}}\widehat{\pi}_{t-1}^H + \beta\frac{\phi_{C_{H1}} + \phi_{C_{H2}}}{\phi_{C_{H1}} + (1+\beta)\,\phi_{C_{H2}}}E_t\widehat{\pi}_{t+1}^H \\
\widehat{\pi}_t^F = & -\frac{\varepsilon_F}{\phi_{C_{F1}} + (1+\beta)\,\phi_{C_{F2}}}\widehat{\varepsilon}_{F,t} + \frac{\varepsilon_F(\varepsilon_F - 1)}{\phi_{C_{F1}} + (1+\beta)\,\phi_{C_{F2}}}\left(\widehat{Q}_t - \widehat{p}_{F,t}\right) \\
& + \frac{\phi_{C_{F2}}}{\phi_{C_{F1}} + (1+\beta)\,\phi_{C_{F2}}}\widehat{\pi}_{t-1}^F + \beta\frac{\phi_{C_{F1}} + \phi_{C_{F2}}}{\phi_{C_{F1}} + (1+\beta)\,\phi_{C_{F2}}}E_t\widehat{\pi}_{t+1}^F.
\end{aligned}$$

In the estimation I set the priors in such a way that the mode is close to fifty-fifty for the gross coefficients for expected future inflation and lagged inflation. Then, the higher the estimate for the ϕ_{C_1}s and the lower the estimates for the ϕ_{C_2}s, the more weight is put on

Table 3: Estimation results. Benchmark model

Parameter	Description	Prior distribution			Posterior distribution				
		Type	Mean	S.D.	Mode	S.D.	Mean	5%	95%
$\phi_{C_{H1}}$	Cost. steady state	Inv. gam.	0.150	inf.	0.211	0.075	0.261	0.130	0.386
$\phi_{C_{H2}}$	Cost. prev. period	Inv. gam.	0.075	inf.	0.025	0.007	0.033	0.017	0.050
$\phi_{C_{F1}}$	Cost. steady state	Inv. gam.	0.150	inf.	1.296	0.362	1.506	0.785	2.208
$\phi_{C_{F2}}$	Cost. prev. period	Inv. gam.	0.075	inf.	0.034	0.014	0.062	0.018	0.110
ρ_y	Pers. Y	Beta	0.5	0.2	0.729	0.034	0.739	0.684	0.795
ρ_b	Pers. b	Beta	0.5	0.2	0.870	0.023	0.866	0.830	0.904
ρ_G	Pers. G	Beta	0.5	0.2	0.987	0.006	0.986	0.976	0.996
ρ_{ε_H}	Pers. ε_H	Beta	0.5	0.2	0.549	0.186	0.491	0.246	0.730
ρ_{ε_F}	Pers. ε_F	Beta	0.5	0.2	0.286	0.113	0.287	0.109	0.459
stderr ξ^Y		Inv. gam.	0.02	inf.	0.009	0.001	0.010	0.008	0.011
stderr ξ^b		Inv. gam.	0.01	inf.	0.003	0.001	0.004	0.003	0.005
stderr ξ^G		Inv. gam.	0.012	inf.	0.018	0.001	0.018	0.016	0.020
stderr ξ^r		Inv. gam.	0.0025	inf.	0.004	0.0003	0.004	0.003	0.004
stderr ξ^{ε_H}		Inv. gam.	0.05	inf.	0.067	0.022	0.082	0.049	0.113
stderr ξ^{ε_F}		Inv. gam.	0.05	inf.	0.744	0.222	0.458	0.186	0.732

$$\widehat{\pi}_t^H = -0.018\widehat{\varepsilon}_{H,t} + 0.092\left(\widehat{w}_t - \widehat{Z}_t - \widehat{p}_{H,t}\right) + 0.101\widehat{\pi}_{t-1}^H + 0.893E_t\widehat{\pi}_{t+1}^H$$

$$\widehat{\pi}_t^F = -0.004\widehat{\varepsilon}_{F,t} + 0.018\left(\widehat{Q}_t - \widehat{p}_{F,t}\right) + 0.038\widehat{\pi}_{t-1}^F + 0.955E_t\widehat{\pi}_{t+1}^F$$

Draws	1.5 mill.
Log d.d.	1548
Acc.rate	0.33

expected future inflation. Note that, when coding up the model for estimation, I multiply the cost parameters by a thousand, and thus the results must be read in thousands.

From Table 3 we see that there is a lot of weight put on the forward term in the Phillips curves for both domestic and imported inflation. The coefficients on the lead term in the domestic and import price inflation curves are 0.89 and 0.96, respectively. The lag term coefficients are 0.10 and 0.04 for domestic inflation and import inflation, respectively.

5.2 Classic model

When estimating the model with the purely forward looking New Keynesian Phillips curves, the ϕ_{C2}s, are set to zero. I then scale the priors for the ϕ_{C1}s, so that the priors for the gross parameters on marginal costs and the mark-up shocks have about the same mean as the priors for the benchmark model. The Phillips curves are

$$\widehat{\pi}_t^H = -\frac{\varepsilon_H}{\phi_{C_{H1}}}\widehat{\varepsilon}_{H,t} + \frac{\varepsilon_H(\varepsilon_H - 1)}{\phi_{C_{H1}}}\left(\widehat{w}_t - \widehat{Z}_t - \widehat{p}_{H,t}\right) + \beta E_t\widehat{\pi}_{t+1}^H$$
$$\widehat{\pi}_t^F = -\frac{\varepsilon_F}{\phi_{C_{F1}}}\widehat{\varepsilon}_{F,t} + \frac{\varepsilon_F(\varepsilon_F - 1)}{\phi_{C_{F1}}}\left(\widehat{Q}_t - \widehat{p}_{F,t}\right) + \beta E_t\widehat{\pi}_{t+1}^F.$$

The results from the estimation are given in Table 4.We see that there are only minor

Table 4: Estimation results. Classic model

Parameter	Description	Prior distribution			Posterior distribution				
		Type	Mean	S.D.	Mode	S.D.	Mean	5%	95%
$\phi_{C_{H1}}$	Cost. steady state	Inv. gam.	0.3	0.5	0.187	0.052	0.234	0.129	0.335
$\phi_{C_{H2}}$	Cost. prev. period	Set	0	–					
$\phi_{C_{F1}}$	Cost. steady state	Inv. gam.	0.3	0.5	1.305	0.356	1.513	0.813	2.192
$\phi_{C_{F2}}$	Cost. prev. period	Set	0	–					
ρ_y	Pers. Y	Beta	0.5	0.2	0.723	0.035	0.732	0.675	0.788
ρ_b	Pers. b	Beta	0.5	0.2	0.872	0.022	0.868	0.832	0.905
ρ_G	Pers. G	Beta	0.5	0.2	0.986	0.006	0.985	0.975	0.996
ρ_{ε_H}	Pers. ε_H	Beta	0.5	0.2	0.674	0.133	0.609	0.395	0.824
ρ_{ε_F}	Pers. ε_F	Beta	0.5	0.2	0.299	0.115	0.314	0.130	0.490
stderr ξ^Y		Inv. gam.	0.02	inf.	0.009	0.001	0.010	0.008	0.011
stderr ξ^b		Inv. gam.	0.01	inf.	0.003	0.001	0.003	0.002	0.004
stderr ξ^G		Inv. gam.	0.012	inf.	0.018	0.001	0.018	0.016	0.020
stderr ξ^r		Inv. gam.	0.0025	inf.	0.004	0.0003	0.004	0.003	0.004
stderr ξ^{ε_H}		Inv. gam.	0.05	inf.	0.049	0.013	0.062	0.036	0.086
stderr ξ^{ε_F}		Inv. gam.	0.05	inf.	0.720	0.221	0.831	0.413	1.239

$$\widehat{\pi}_t^H = -0.026\widehat{\varepsilon}_{H,t} + 0.128\left(\widehat{w}_t - \widehat{Z}_t - \widehat{p}_{H,t}\right) + \beta E_t \widehat{\pi}_{t+1}^H$$

$$\widehat{\pi}_t^F = -0.004\widehat{\varepsilon}_{F,t} + 0.020\left(\widehat{Q}_t - \widehat{p}_{F,t}\right) + \beta E_t \widehat{\pi}_{t+1}^F$$

Draws	1.5 mill
Log d.d.	1554
Acc.rate	0.37

changes in the estimates for the coefficients on marginal costs and mark-up shocks in the import curve. In the domestic curve, the estimate for the coefficient on the mark-up shock increases (in absolute terms) from 0.018 to 0.026. But in addition, the estimate for the standard error of the shock decreases from 0.082 to 0.062, so the estimated gross shock stays about unchanged. The estimate for the coefficient on marginal cost in the domestic curve increases from 0.092 to 0.128.

5.3 Restricted hybrid version

In this version of the model, the Phillips curves are based on the assumption of no adjustment costs related to price changes relative to steady state inflation (that is, the ϕ_{C_1}s are set to zero). The result is hybrid curves in which both past and expected future inflation plays a part in the pricing decisions, but with equal weight of about one half on both terms. These

Table 5: Estimation results. Restricted hybrid model

Parameter	Description	Prior distribution			Posterior distribution				
		Type	Mean	S.D.	Mode	S.D.	Mean	5%	95%
$\phi_{C_{H1}}$	Cost. steady state	Set	0	–					
$\phi_{C_{H2}}$	Cost. prev. period	Inv. gam.	0.15	inf.	0.08	0.021	0.223	0.056	0.396
$\phi_{C_{F1}}$	Cost. steady state	Set	0	–					
$\phi_{C_{F2}}$	Cost. prev. period	Inv. gam.	0.15	inf.	0.705	0.183	0.812	0.433	1.180
ρ_y	Pers. Y	Beta	0.5	0.2	0.781	0.053	0.735	0.643	0.830
ρ_b	Pers. b	Beta	0.5	0.2	0.863	0.025	0.853	0.811	0.894
ρ_G	Pers. G	Beta	0.5	0.2	0.990	0.006	0.982	0.969	0.996
ρ_{ε_H}	Pers. ε_H	Beta	0.5	0.2	0.810	0.095	0.414	0.043	0.816
ρ_{ε_F}	Pers. ε_F	Beta	0.5	0.2	0.052	0.042	0.079	0.007	0.148
stderr ξ^Y		Inv. gam.	0.02	inf.	0.009	0.001	0.010	0.008	0.011
stderr ξ^b		Inv. gam.	0.01	inf.	0.003	0.001	0.003	0.002	0.004
stderr ξ^G		Inv. gam.	0.012	inf.	0.018	0.001	0.018	0.016	0.020
stderr ξ^r		Inv. gam.	0.0025	inf.	0.004	0.0003	0.004	0.003	0.004
stderr ξ^{ε_H}		Inv. gam.	0.05	inf.	0.059	0.010	0.130	0.047	0.216
stderr ξ^{ε_F}		Inv. gam.	0.05	inf.	0.747	0.169	0.845	0.492	1.190

$\widehat{\pi}_t^H = -0.014\widehat{\varepsilon}_{H,t} + 0.067\left(\widehat{w}_t - \widehat{Z}_t - \widehat{p}_{H,t}\right) + 0.5\widehat{\pi}_{t-1}^H + 0.5E_t\widehat{\pi}_{t+1}^H$	Draws	1.5 mill.
	Log d.d.	1517
$\widehat{\pi}_t^F = -0.004\widehat{\varepsilon}_{F,t} + 0.019\left(\widehat{Q}_t - \widehat{p}_{F,t}\right) + 0.5\widehat{\pi}_{t-1}^F + 0.5E_t\widehat{\pi}_{t+1}^F$	Acc.rate	0.40

are the Phillips curves used in Norges Bank's NEMO (Brubakk et al., 2006).

$$
\begin{aligned}
\widehat{\pi}_t^H =& -\frac{\varepsilon_H}{(1+\beta)\,\phi_{C_{H2}}}\widehat{\varepsilon}_{H,t} + \frac{\varepsilon_H(\varepsilon_H - 1)}{(1+\beta)\,\phi_{C_{H2}}}\left(\widehat{w}_t - \widehat{Z}_t - \widehat{p}_{H,t}\right) \\
&+ \frac{1}{1+\beta}\widehat{\pi}_{t-1}^H + \frac{\beta}{1+\beta}E_t\widehat{\pi}_{t+1}^H \\
\widehat{\pi}_t^F =& -\frac{\varepsilon_F}{(1+\beta)\,\phi_{C_{F2}}}\widehat{\varepsilon}_{F,t} + \frac{\varepsilon_F(\varepsilon_F - 1)}{(1+\beta)\,\phi_{C_{F2}}}\left(\widehat{Q}_t - \widehat{p}_{F,t}\right) \\
&+ \frac{1}{1+\beta}\widehat{\pi}_{t-1}^F + \frac{\beta}{1+\beta}E_t\widehat{\pi}_{t+1}^F
\end{aligned}
$$

The estimation results are given in Table 5. The plots of the posteriors in Appendix A.3 seem to indicate some sort of identification problems for the parameters in the domestic Phillips curve. This could point to an insufficient number of draws, but is probably more likely to reflect a genuine identification problem. Compared to the benchmark model, the estimate for the coefficient on marginal costs in the domestic curve reduces from 0.092 to 0.067. The other gross parameters in the two curves change only marginally.

5.4 Models with looser restrictions on the Phillips curves

As we saw in Section 2.2.2, the Calvo assumption on pricing with some rule of thumb behavior can give rise to a hybrid Phillips curve where the coefficients on both the lead and lag term can vary from zero to one. I estimate two versions of the model where this applies: one where the sum of the coefficients on lead and lagged inflation is restricted to one (the homogeneous model) and one where the coefficients are estimated freely (but are still restricted to be between zero and one, individually. The non-homogeneous model). I will focus on the gross coefficients on all terms in the curves, so the curves are the homogeneous

$$\begin{aligned}
\widehat{\pi}_t^H &= -\widehat{\varepsilon}_{H,t} + \gamma_{m,H}\left(\widehat{w}_t - \widehat{Z}_t - \widehat{p}_{H,t}\right) \\
&\quad + \gamma_{b,H}\widehat{\pi}_{t-1}^H + (1-\gamma_{b,H})\, E_t\widehat{\pi}_{t+1}^H \\
\widehat{\pi}_t^F &= -\widehat{\varepsilon}_{F,t} + \gamma_{m,F}\left(\widehat{Q}_t - \widehat{p}_{F,t}\right) \\
&\quad + \gamma_{b,F}\widehat{\pi}_{t-1}^F + (1-\gamma_{b,F})\, E_t\widehat{\pi}_{t+1}^F,
\end{aligned}$$

and the non-homogeneous

$$\begin{aligned}
\widehat{\pi}_t^H &= -\widehat{\varepsilon}_{H,t} + \gamma_{m,H}\left(\widehat{w}_t - \widehat{Z}_t - \widehat{p}_{H,t}\right) \\
&\quad + \gamma_{b,H}\widehat{\pi}_{t-1}^H + \gamma_{f,H}E_t\widehat{\pi}_{t+1}^H \\
\widehat{\pi}_t^F &= -\widehat{\varepsilon}_{F,t} + \gamma_{m,F}\left(\widehat{Q}_t - \widehat{p}_{F,t}\right) \\
&\quad + \gamma_{b,F}\widehat{\pi}_{t-1}^F + \gamma_{f,F}E_t\widehat{\pi}_{t+1}^F.
\end{aligned}$$

As priors for the inflation term coefficients in both models, I choose a beta distribution with mean 0.5 and a standard deviation of 0.2. This is a relatively loose prior. I set the priors for the coefficients on marginal costs and for the mark-up shocks so that their mean correspond to the priors in the previous models. The estimation results are given in Table 6 and Table 7. We see that expected future inflation is dominant in both models and for both imported and domestic inflation. For the homogeneous model the coefficient estimates for the lead terms in domestic and imported inflation are 0.91 and 0.84, respectively. The non-homogeneous model gives estimates of 0.83 and 0.49 for the lead term coefficients and 0.12 and 0.17 for the lagged term coefficients on domestic and imported inflation, respectively. Notice also that the sum of the coefficients on the lead term and the lagged term in domestic inflation in the non-homogeneous model is close to unity (0.95). For imported inflation the

Table 6: Estimation results. Homogeneous model

Parameter	Description	Prior distribution			Posterior distribution				
		Type	Mean	S.D.	Mode	S.D.	Mean	5%	95%
$\gamma_{m,H}$	Marg. cost	Gamma	0.1	0.5	0.081	0.028	0.093	0.039	0.148
$\gamma_{m,F}$	Marg. cost	Gamma	0.1	0.5	0.009	0.003	0.011	0.005	0.018
$\gamma_{b,H}$	Lag term	Beta	0.5	0.2	0.068	0.052	0.095	0.012	0.173
$(1-\gamma_{b,H})$	Lead term								
$\gamma_{b,F}$	Lag term	Beta	0.5	0.2	0.190	0.078	0.164	0.050	0.272
$(1-\gamma_{b,F})$	Lead term								
ρ_y	Pers. Y	Beta	0.5	0.2	0.752	0.033	0.747	0.689	0.804
ρ_b	Pers. b	Beta	0.5	0.2	0.884	0.022	0.874	0.838	0.910
ρ_G	Pers. G	Beta	0.5	0.2	0.989	0.006	0.986	0.976	0.996
$\rho_{\varepsilon H}$	Pers. ε_H	Beta	0.5	0.2	0.463	0.174	0.480	0.214	0.753
$\rho_{\varepsilon H}$	Pers. ε_F	Beta	0.5	0.2	0.097	0.082	0.162	0.019	0.295
stderr ξ^Y		Inv. gam.	0.02	inf.	0.009	0.001	0.009	0.008	0.011
stderr ξ^b		Inv. gam.	0.01	inf.	0.003	0.0004	0.003	0.002	0.004
stderr ξ^G		Inv. gam.	0.012	inf.	0.018	0.001	0.018	0.016	0.020
stderr ξ^r		Inv. gam.	0.0025	inf.	0.004	0.0003	0.004	0.003	0.004
stderr $\xi^{\varepsilon H}$		Inv. gam.	0.001	inf.	0.002	0.0002	0.002	0.001	0.002
stderr $\xi^{\varepsilon F}$		Inv. gam.	0.001	inf.	0.003	0.0003	0.003	0.003	0.004

$$\widehat{\pi}_t^H = -\widehat{\varepsilon}_{H,t} + 0.093\left(\widehat{w}_t - \widehat{Z}_t - \widehat{p}_{H,t}\right) + 0.095\widehat{\pi}_{t-1}^H + 0.905E_t\widehat{\pi}_{t+1}^H$$

$$\widehat{\pi}_t^F = -\widehat{\varepsilon}_{F,t} + 0.011\left(\widehat{Q}_t - \widehat{p}_{F,t}\right) + 0.164\widehat{\pi}_{t-1}^F + 0.836E_t\widehat{\pi}_{t+1}^F$$

Draws	1.5 mill.
Log d.d.	1547
Acc.rate	0.32

sum of the coefficients is a bit smaller (0.66).

5.5 Model comparison

As described in Section 4, we can compare the models' predictive abilities by comparing the log data density of a model to another. This is, unfortunately, not as straightforward as it sounds. When we change a model, we may have a different number of parameters to estimate. This could, ceteris paribus, alter the log data density in favor of the model with fewer parameters to be estimated. Moreover, the choice of prior densities also affect the data density. This is why I tried to keep the priors for the gross parameters on the different terms in the Phillips curves as equal as possible for the different models.

We see from tables 3 and 5 that the benchmark model has higher log data density (1548) than the hybrid model with the restriction of fifty-fifty on the lead and lag term (1517). This, in spite the fact that the restricted model has fewer estimated parameters. This suggests that expected future inflation is an important variable in the price setting rules

The model with purely forward looking Phillips curves has higher log data density (1554)

Table 7: Estimation results. Non-homogeneous model

Parameter	Description	Prior distribution			Posterior distribution				
		Type	Mean	S.D.	Mode	S.D.	Mean	5%	95%
$\gamma_{m,H}$	Marg. cost	Gamma	0.1	0.5	0.067	0.023	0.083	0.034	0.130
$\gamma_{m,F}$	Marg. cost	Gamma	0.1	0.5	0.021	0.017	0.038	0.017	0.062
$\gamma_{b,H}$	Lag term	Beta	0.5	0.2	0.107	0.070	0.118	0.024	0.206
$\gamma_{f,H}$	Lead term	Beta	0.5	0.2	0.844	0.070	0.831	0.726	0.943
$\gamma_{b,F}$	Lag term	Beta	0.5	0.2	0.190	0.080	0.165	0.048	0.279
$\gamma_{f,F}$	Lead term	Beta	0.5	0.2	0.661	0.197	0.495	0.200	0.792
ρ_y	Pers. Y	Beta	0.5	0.2	0.743	0.036	0.742	0.680	0.805
ρ_b	Pers. b	Beta	0.5	0.2	0.878	0.023	0.870	0.833	0.907
ρ_G	Pers. G	Beta	0.5	0.2	0.989	0.006	0.987	0.978	0.997
$\rho_{\varepsilon H}$	Pers. ε_H	Beta	0.5	0.2	0.345	0.166	0.427	0.156	0.707
$\rho_{\varepsilon H}$	Pers. ε_F	Beta	0.5	0.2	0.087	0.070	0.138	0.018	0.253
stderr ξ^Y		Inv. gam.	0.02	inf.	0.009	0.001	0.010	0.008	0.011
stderr ξ^b		Inv. gam.	0.01	inf.	0.003	0.001	0.003	0.002	0.004
stderr ξ^G		Inv. gam.	0.012	inf.	0.018	0.001	0.018	0.016	0.020
stderr ξ^r		Inv. gam.	0.0025	inf.	0.004	0.0003	0.004	0.003	0.004
stderr $\xi^{\varepsilon H}$		Inv. gam.	0.001	inf.	0.002	0.0002	0.002	0.001	0.002
stderr $\xi^{\varepsilon F}$		Inv. gam.	0.001	inf.	0.003	0.0004	0.004	0.003	0.004

$$\widehat{\pi}_t^H = -\widehat{\varepsilon}_{H,t} + 0.083\left(\widehat{w}_t - \widehat{Z}_t - \widehat{p}_{H,t}\right) + 0.118\widehat{\pi}_{t-1}^H + 0.831 E_t \widehat{\pi}_{t+1}^H$$

$$\widehat{\pi}_t^F = -\widehat{\varepsilon}_{F,t} + 0.038\left(\widehat{Q}_t - \widehat{p}_{F,t}\right) + 0.165\widehat{\pi}_{t-1}^F + 0.495 E_t \widehat{\pi}_{t+1}^F$$

Draws	1.5 mill.
Log d.d.	1544
Acc.rate	0.28

than the benchmark model. This also points in the direction of the importance of expected future inflation. However, notice that the purely forward looking Phillips curve contains fewer estimated parameters than the benchmark model. In addition, the purely forward looking model puts by construction a higher prior weight on expected inflation which seems to be favoured by the data. If I change the priors in the benchmark model to give more weight on the lead term, the result is a log data density close to the one for the purely forward looking model. So the lead term seem to be dominant, but the better fit of the classic model is not necessarily evidence for the exclusion of the lag term.

The two models with fewer restrictions on the coefficients in the Phillips curves both have similar data densities to the benchmark model (1547 for the homogeneous and 1544 for the non-homogeneous). Both of these models point towards expected inflation being dominant in the price setting rules. In addition, since the homogeneous and the non-homogeneous model gives similar densities, we can interpret that as the homogeneity restriction not to be too strong. And as we saw, the sum of the estimates for the coefficients on the two inflation terms in the non-homogeneous model were 0.95 and 0.66 for domestic and imported inflation, respectively.

It could be interesting to compare the estimates for the persistence parameters of the mark-up shocks and the mark-up shocks' standard errors for the different models. This could give indications of whether the persistence in price inflation is intrinsic or extrinsic. However, due to the seemingly problems of identification of these parameters in the restricted model, we should be careful with making inferences.

5.6 Robustness checks

Due to high computational costs of the posterior simulation,[33] I perform the robustness checks based on the calculated mode. None of the results for the benchmark model have shown great deviations between the mode and the posterior mean, so this should be adequate.

By setting the Frisch elasticity parameter to one in the benchmark model, the results for the lead term coefficients change from 0.89 to 0.94 for domestic inflation and from 0.96 to 0.97 for imported inflation. Setting the parameter to 5, as in Adolfson et al. (2007), changes the estimates for the lead terms in domestic and imported inflation to 0.86 and 0.97, respectively.

Changing the relative risk aversion parameter from 1 to 3, decrease the estimates for the lead term coefficient of both domestic and imported inflation to 0.71 and 0.73, respectively.

6 Conclusion

In this thesis I have estimated different versions of the New Keynesian Phillips curve as a part of a standard small open economy DSGE model. The estimation method I have used is Bayesian Maximum Likelihood, and the data are Norwegian quarterly data for the period 1989Q1–2007Q4.

My main findings are that expected future inflation is dominant in the New Keynesian Phillips curve. This result applies to both domestic and imported inflation. When comparing the models, the more flexible the Phillips curves are towards putting weight on expected future inflation, the better they fit the data. A model with a hybrid New Keynesian Phillips curve with a restriction of fifty-fifty on the coefficients on expected future inflation and lagged inflation gives the poorest data fit. The results seem to coincide with the results of Galí and Gertler (1999), Galí et al. (2001), Smets and Wouters (2003), Lindé (2005) and Adolfson et al. (2007) who also find expected future inflation to be dominant in the New Keynesian Phillips curve. However, this runs contrary to the results of Bårdsen et al. (2005) and Boug

[33] 27 and a half hours for the benchmark model with 1.5 million draws.

et al. (2006) who both use Norwegian data and find no role for expected future inflation in the New Keynesian Phillips curve.

I also compare two models with fewer cross restrictions on the gross coefficients in the Phillips curves, one with a homogeneity restriction and one without. The estimation results from these two models also point in the direction of expected future inflation being dominant in the price setting rules. In addition, I find that the homogeneity restriction seems to be justified.

An interesting extension would be to consider the relative forecasting performance of the different specifications of the New Keynesian Phillips curve.

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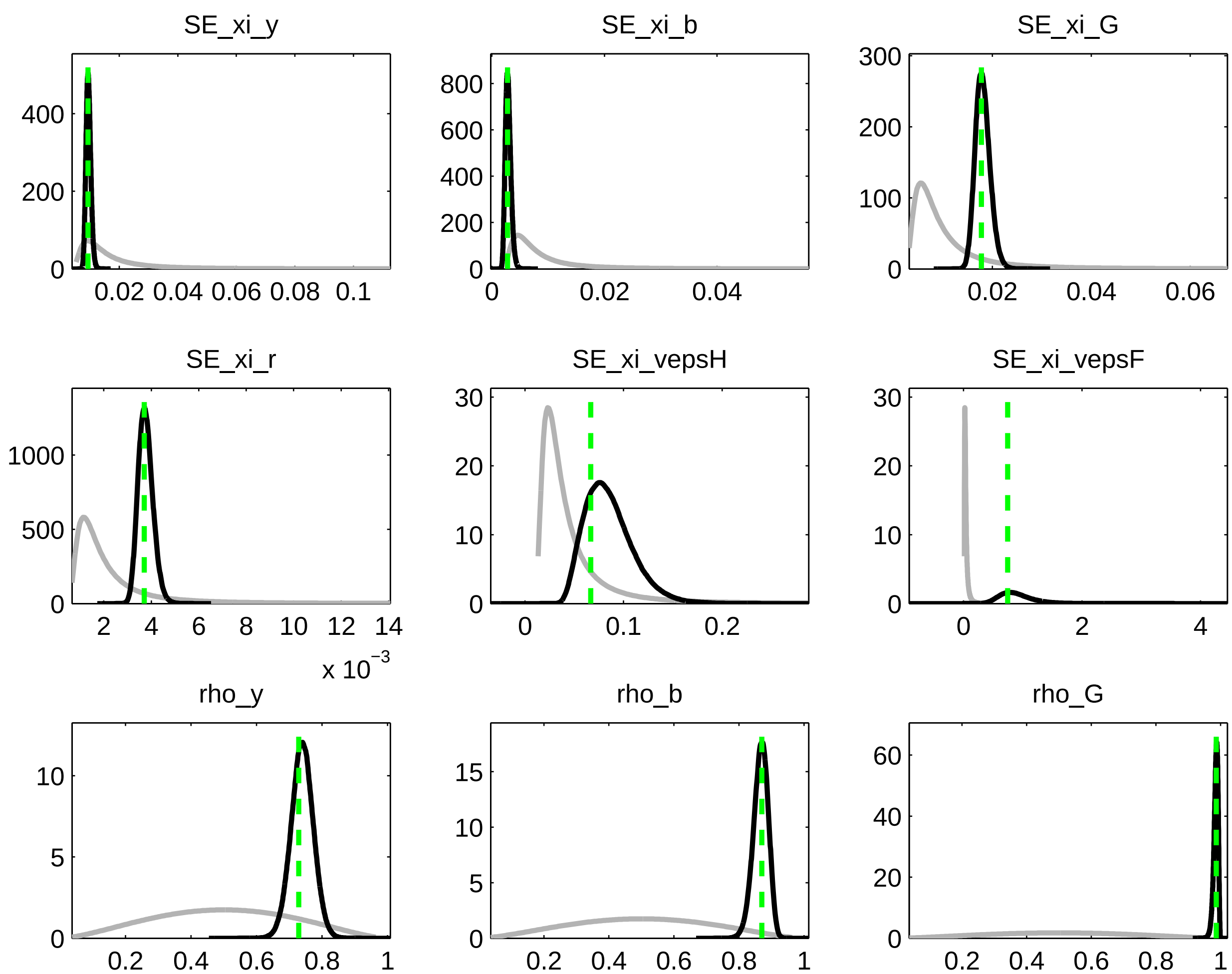
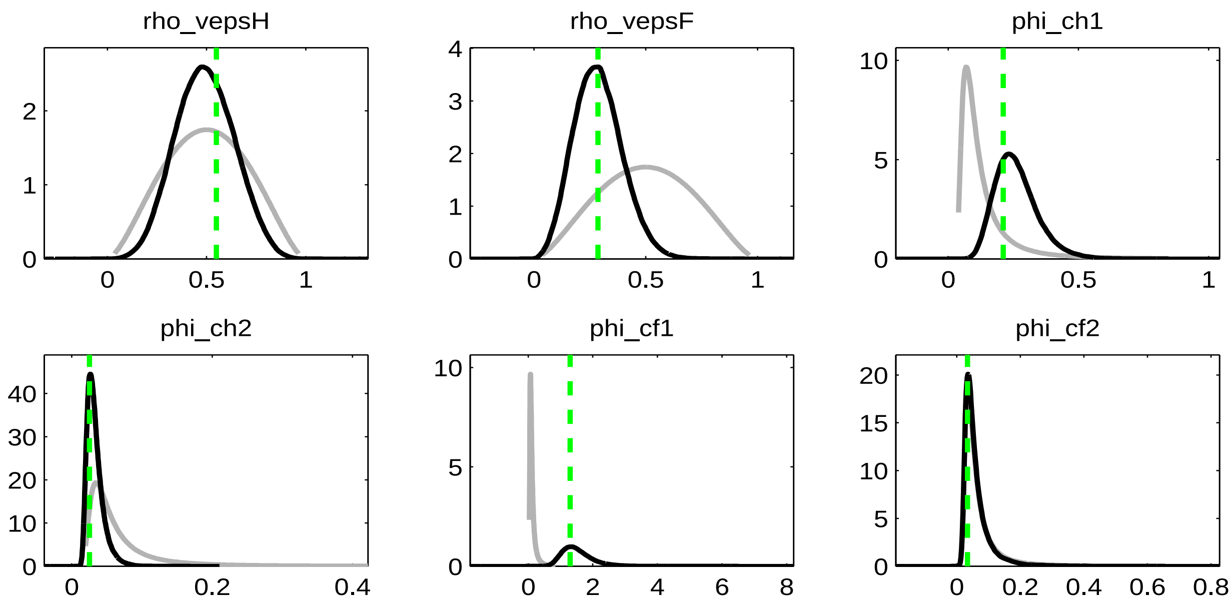
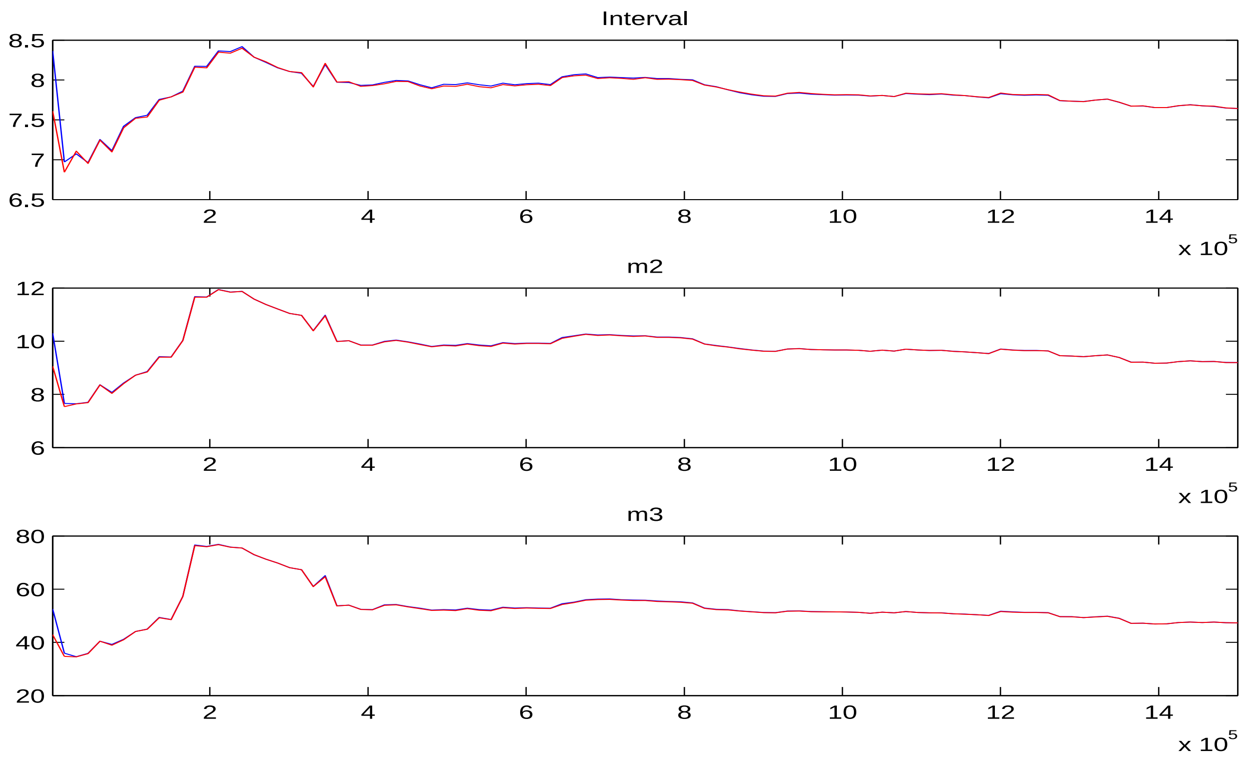
A Estimation output

Two types of figures are printed in this section. The first is a plot of the prior distributions together with the posterior distributions and the modes. The priors are light grey while the posteriors are black. The modes are the (green) dotted lines. The second is a plot of the aggregate convergence diagnostics from the Markov Chains. There is one subsection for each model. Descriptions of the variables' code names are given in Appendix E. Parameter names starting with *SE_* refer to the standard error of the following variable.

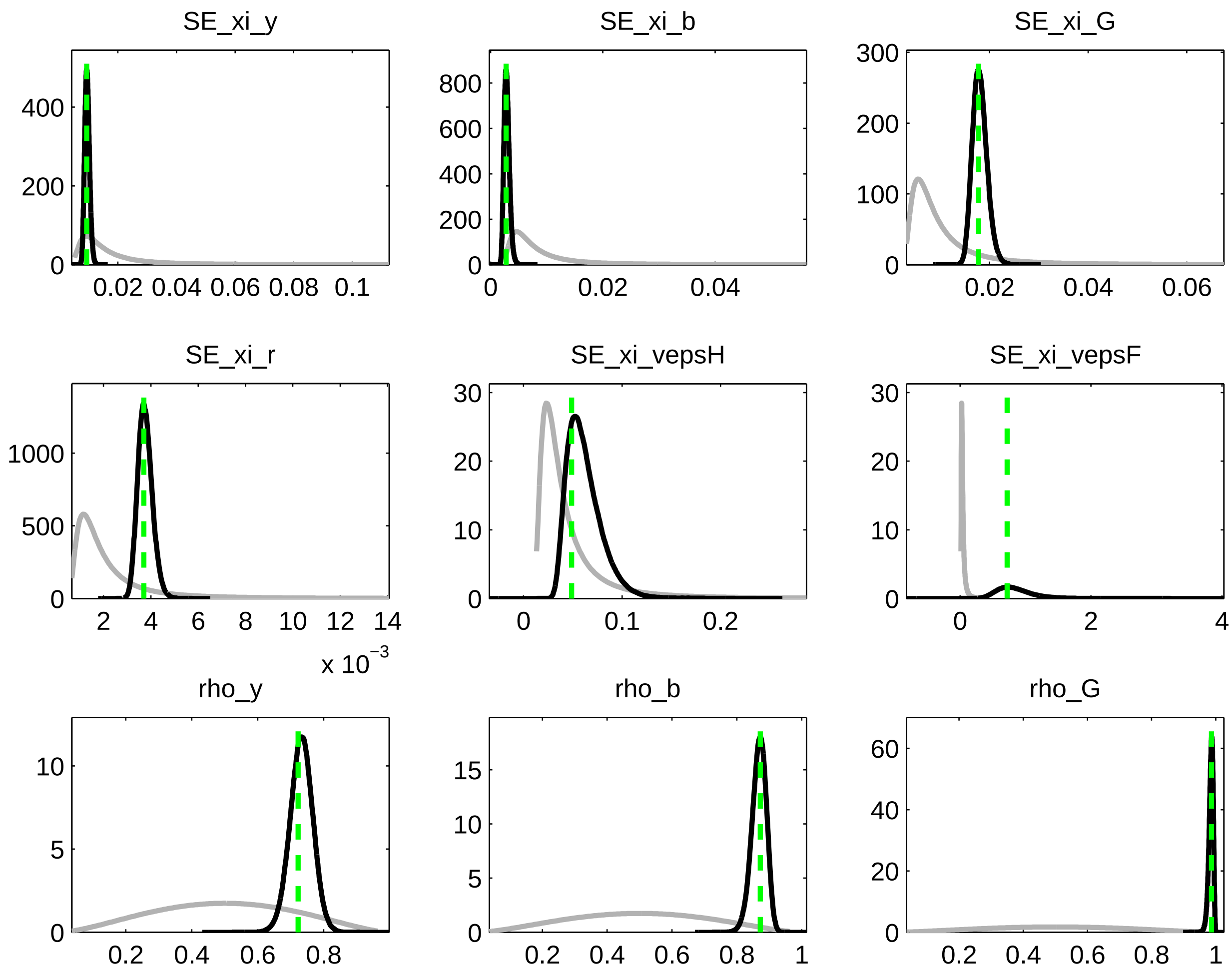
A.1 Benchmark model

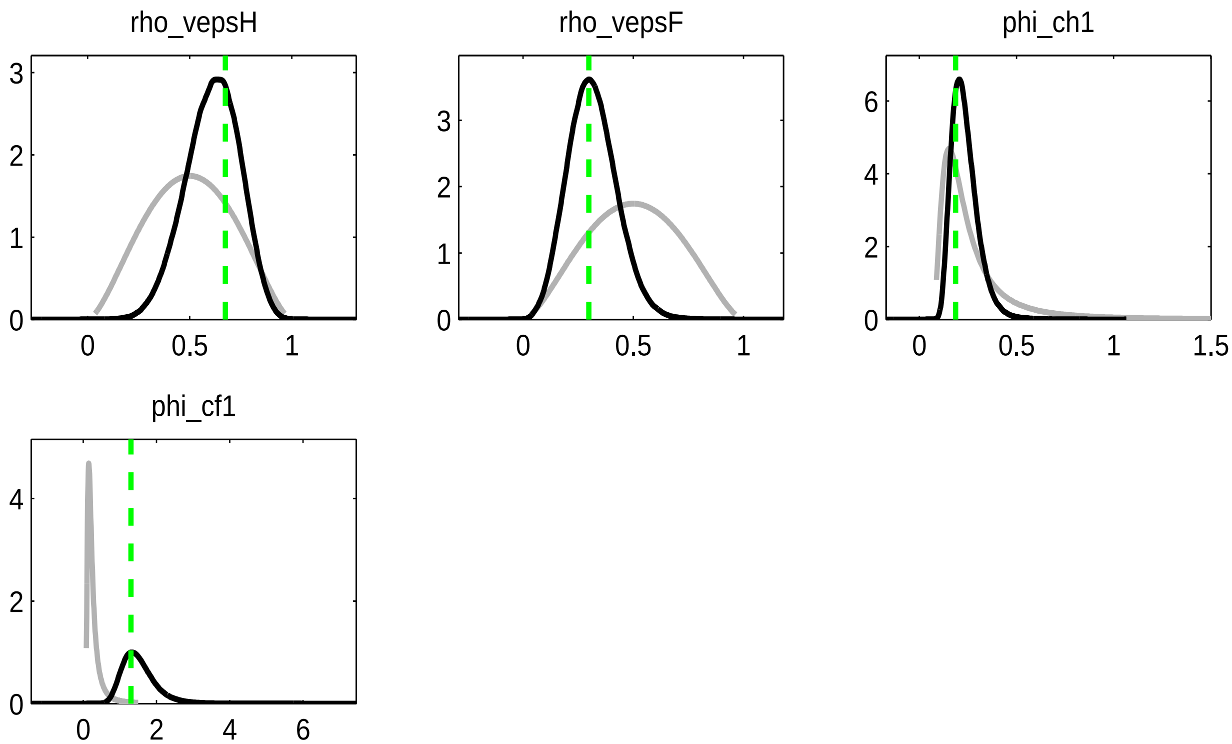


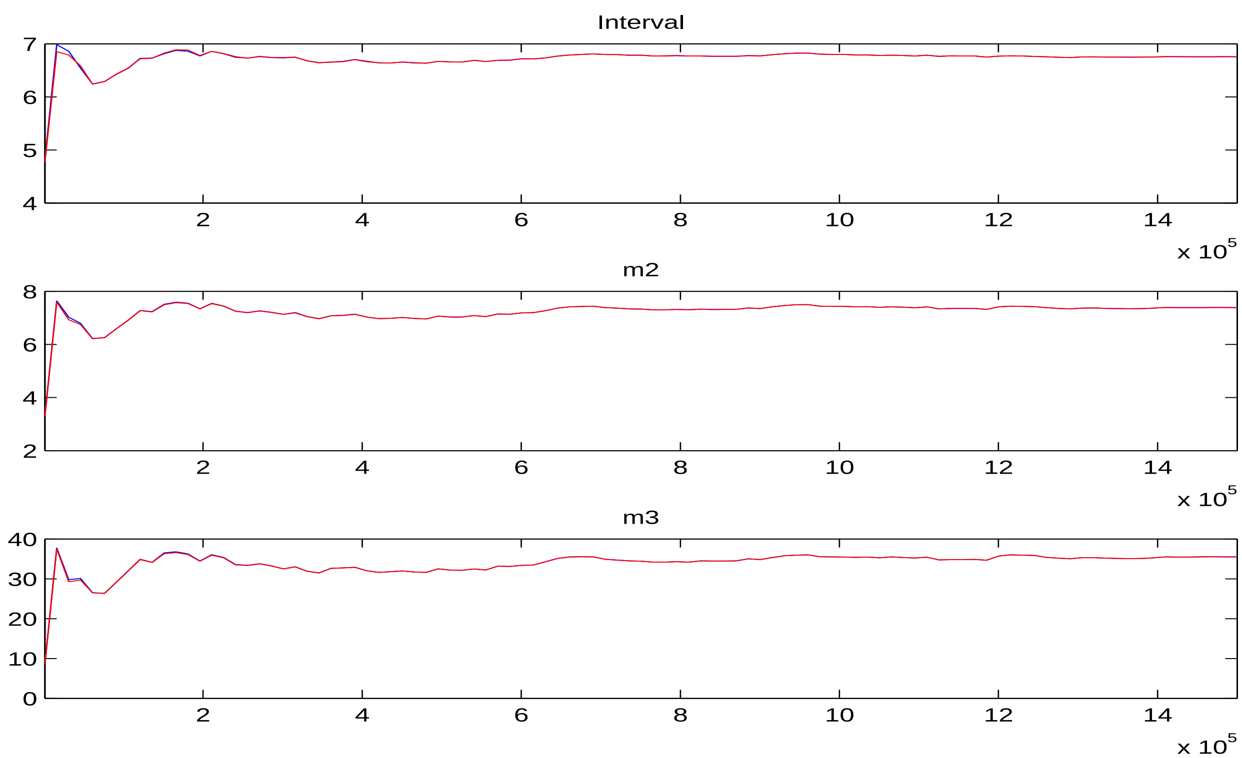

rho_vepsH
rho_vepsF
phi_ch1
phi_ch2
phi_cf1
phi_cf2


Interval
m2
m3
x 10^5

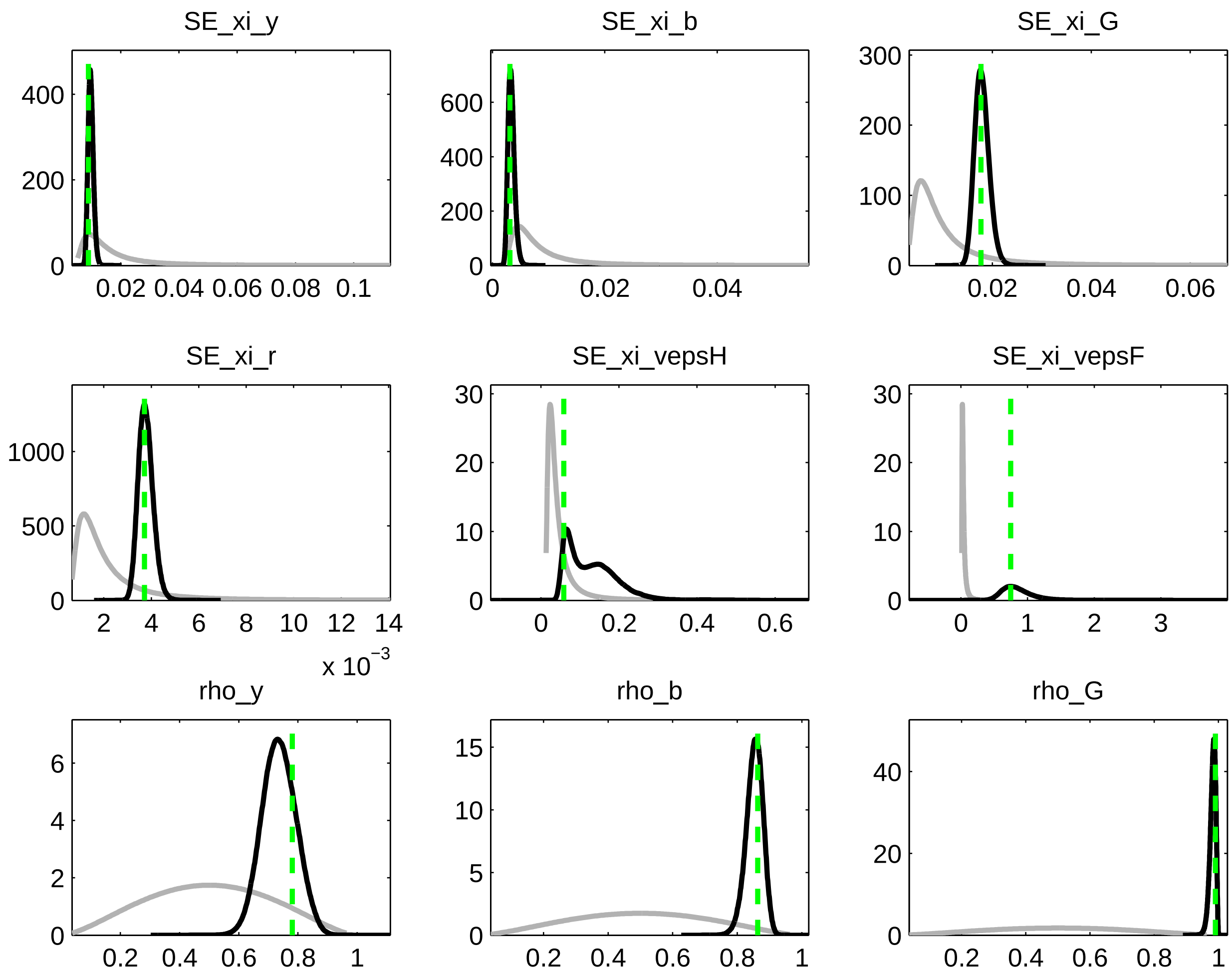
A.2 Classic model

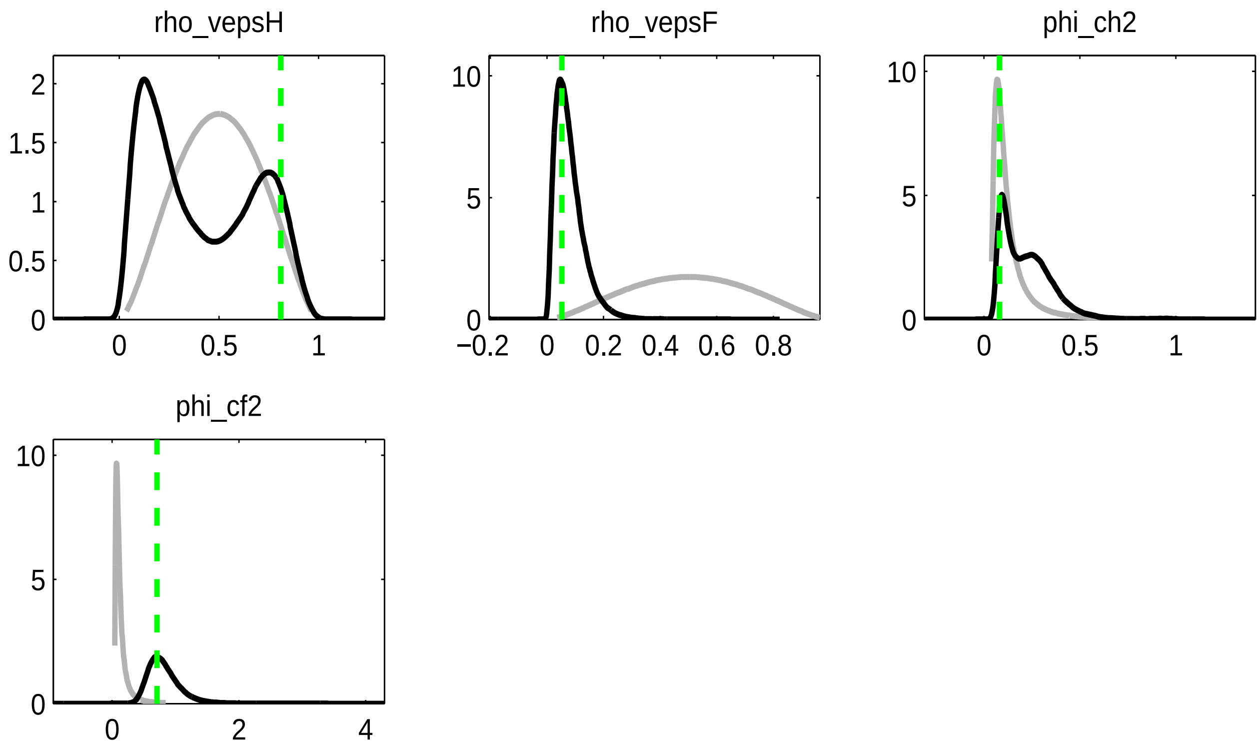


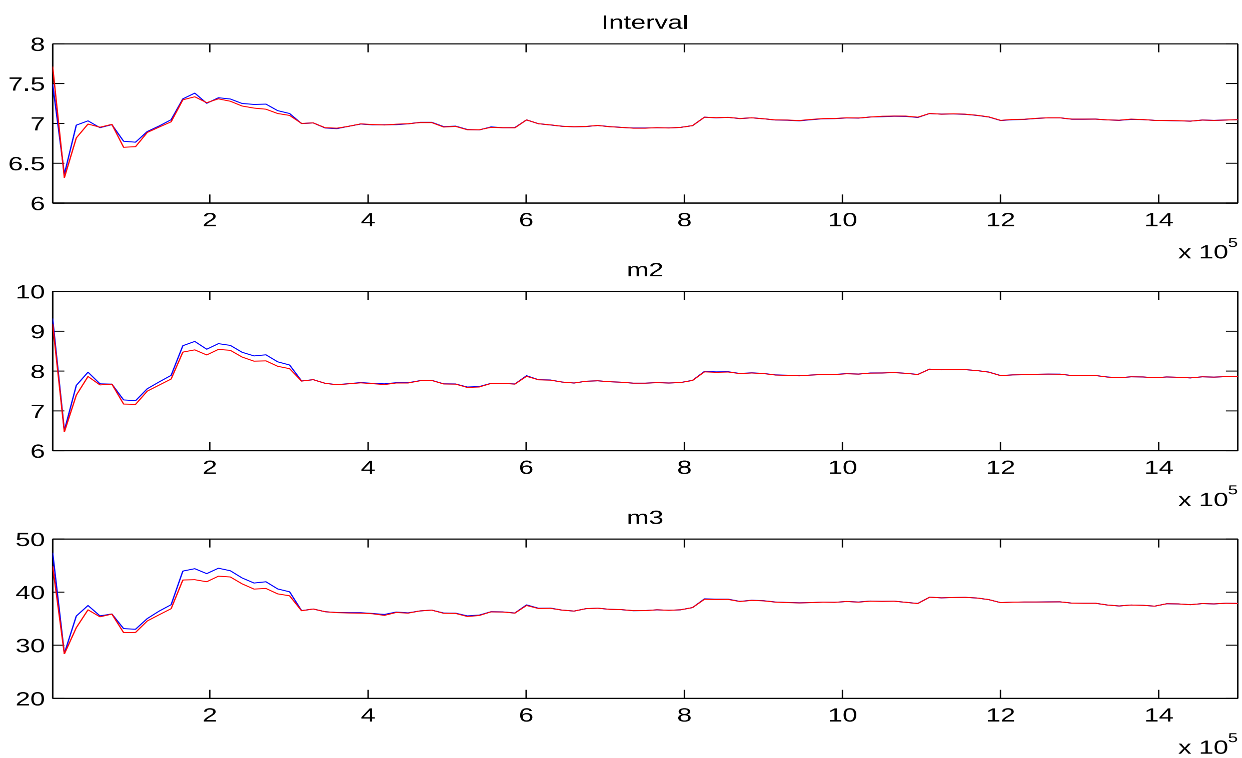

rho_vepsH
rho_vepsF
phi_ch1
phi_cf1


Interval
m2
m3
x 10^5

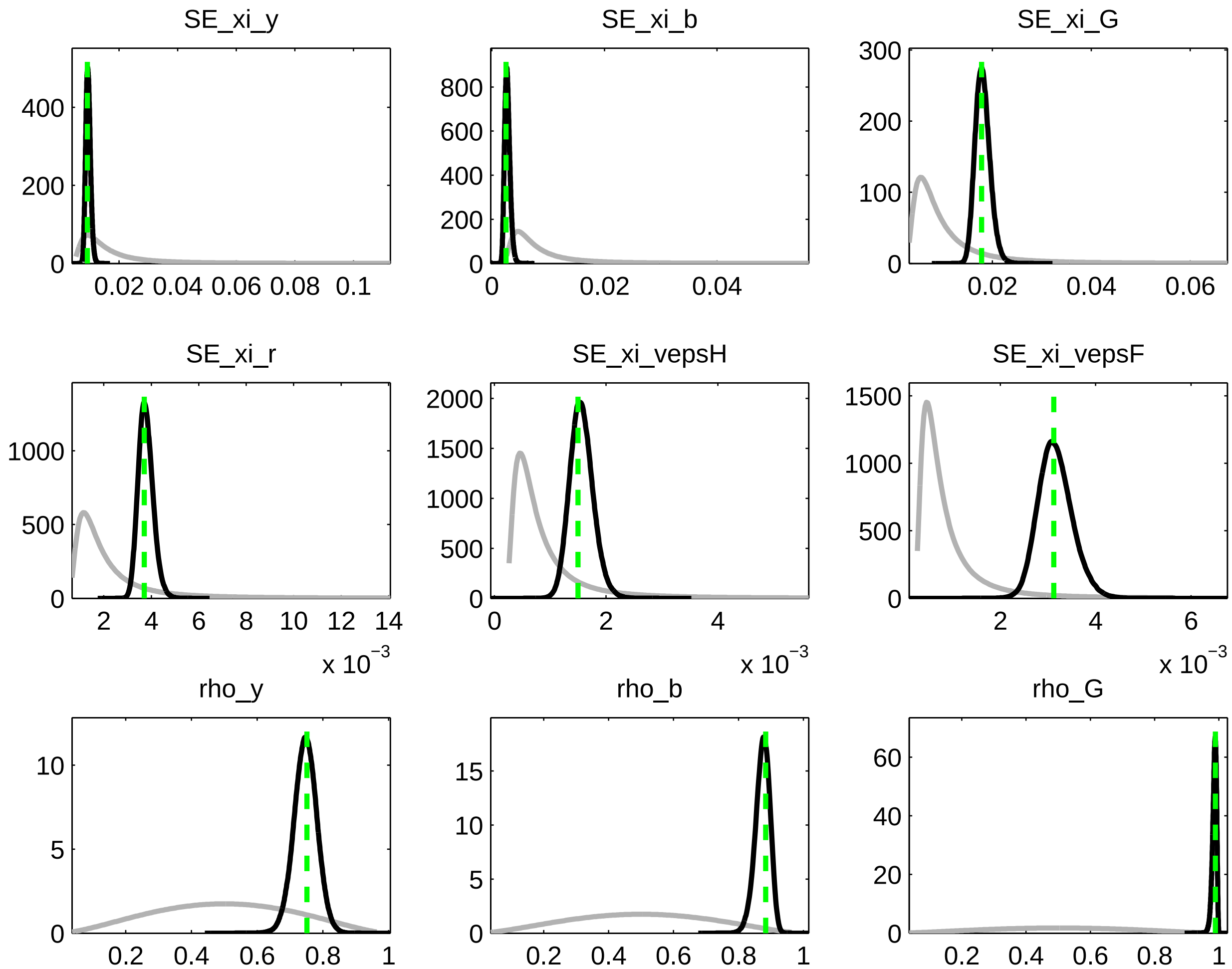
A.3 Restricted model

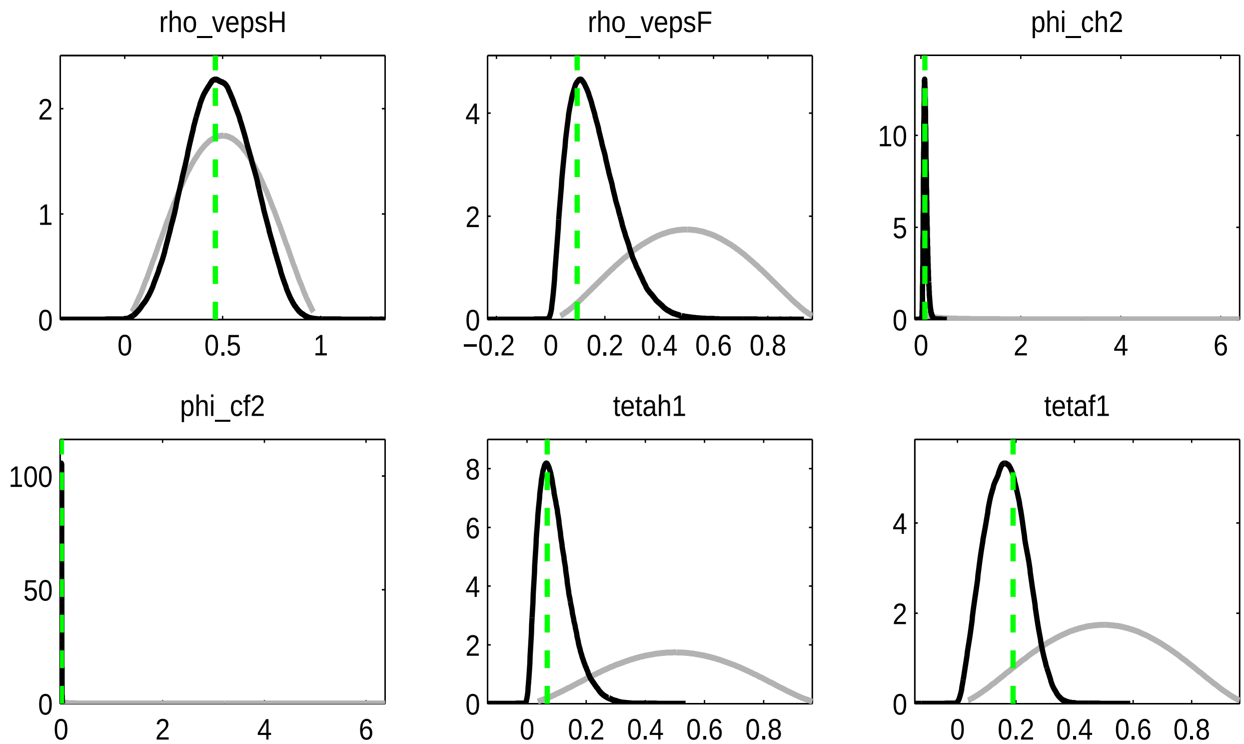


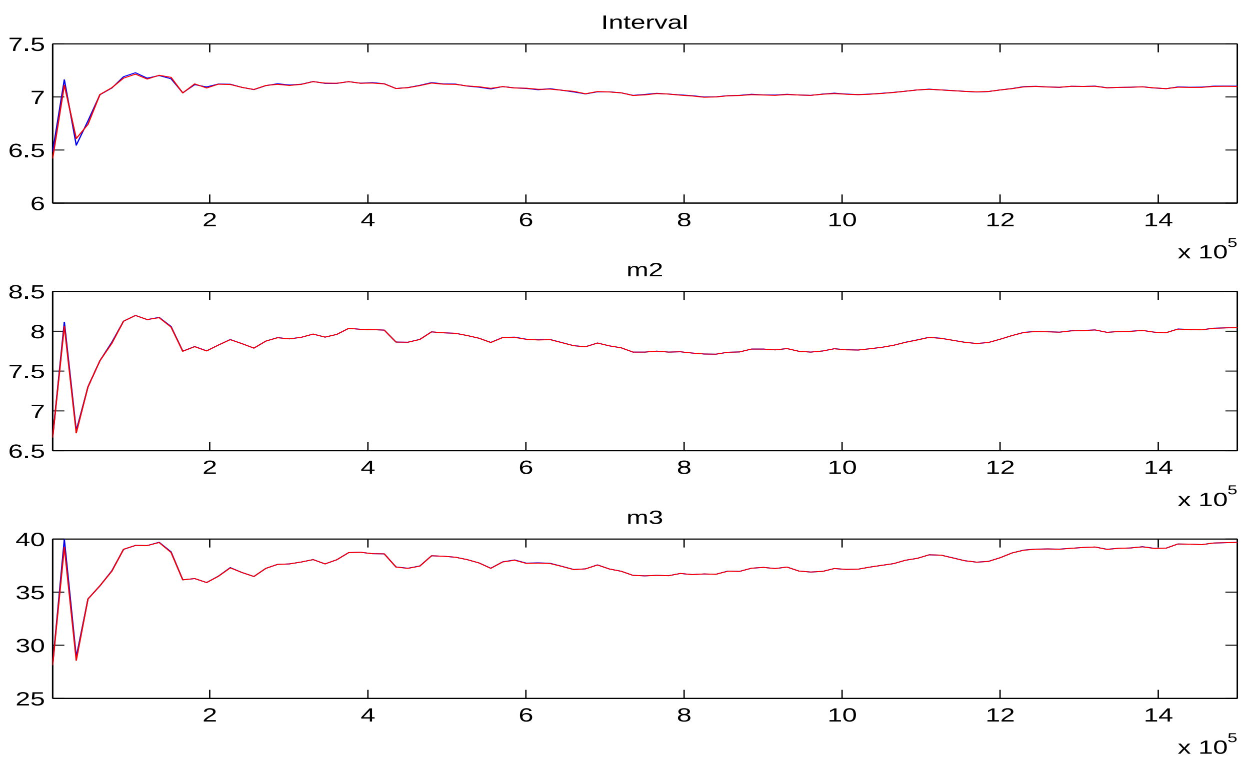

rho_vepsH
rho_vepsF
phi_ch2
phi_cf2


Interval
m2
m3
x 10^5

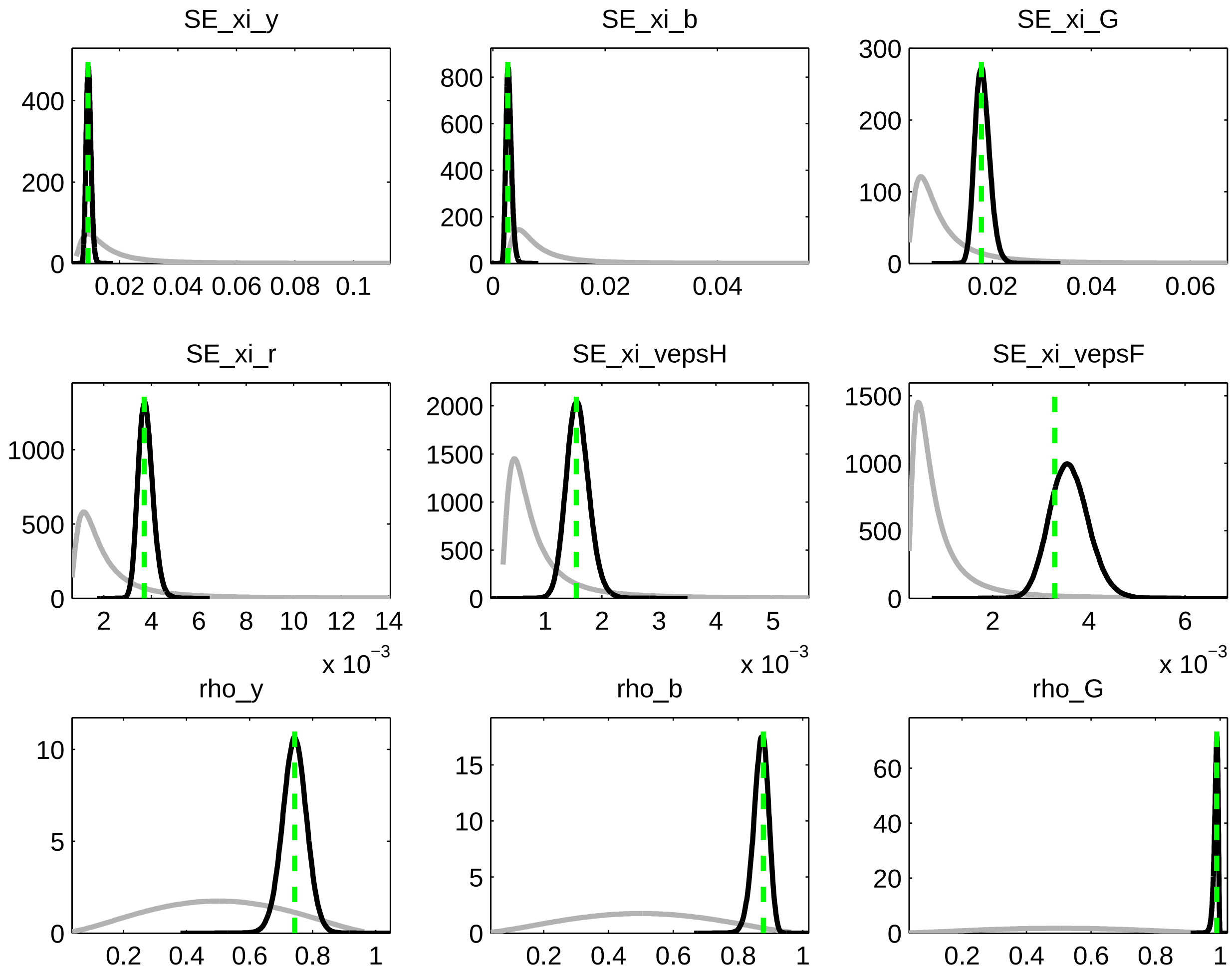
A.4 Homogeneous model

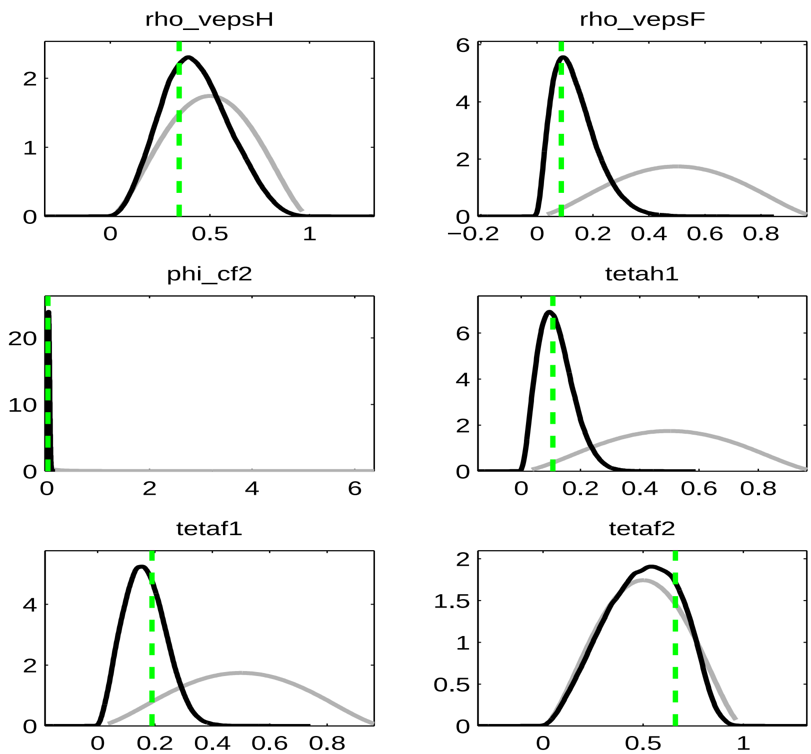


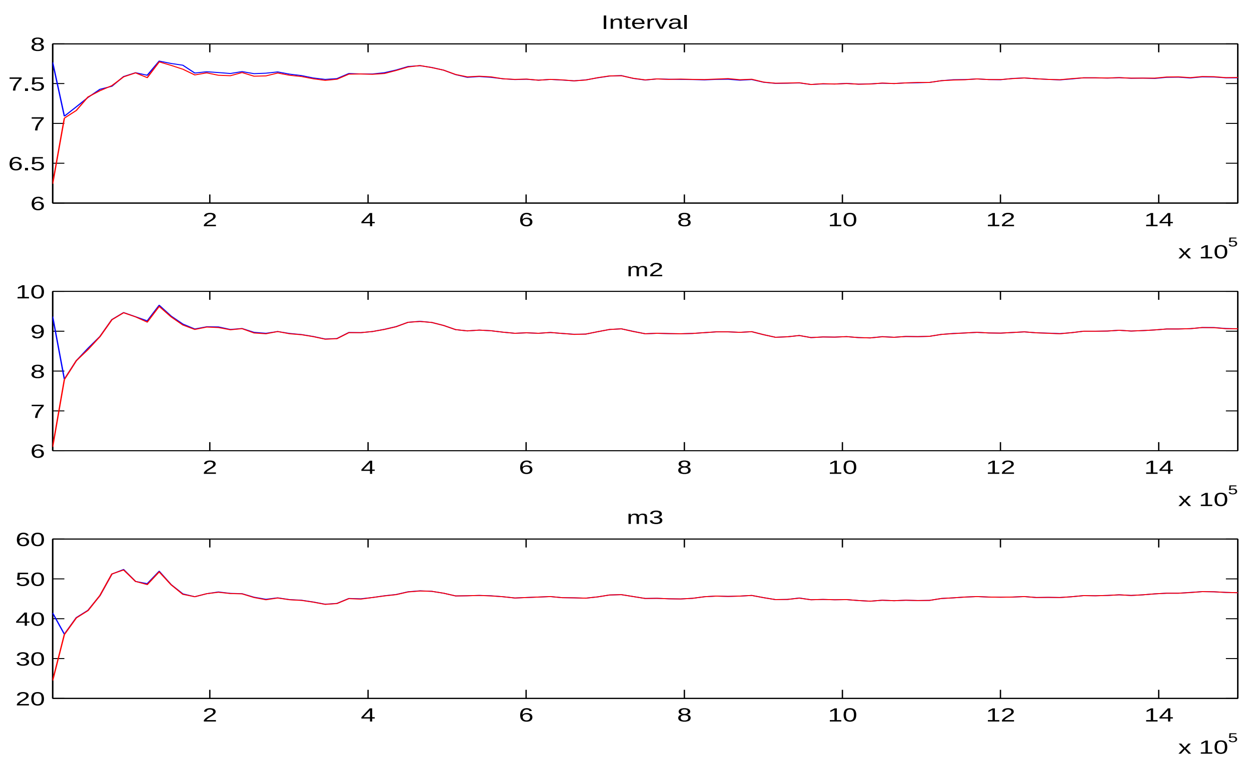

rho_vepsH
rho_vepsF
phi_ch2
phi_cf2
tetah1
tetaf1


Interval
m2
m3
x 10^5

A.5 Non-homogeneous model




rho_vepsH
rho_vepsF
phi_ch2
phi_cf2
tetah1
tetah2
tetaf1
tetaf2


Interval
x 10^5
m2
x 10^5
m3
x 10^5

B Detailed derivation

B.1 Demand

Demand for each type of goods follows from maximization of C_t subject to the nominal budget constraint $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = \overline{B}$:

$$\begin{aligned}
\max_{C_{H,t},C_{F,t}} C_t &= \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\
\text{s.t. } & P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = \overline{B} \\
\mathcal{L} &= \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\
& - \lambda\left(P_{H,t}C_{H,t} + P_{F,t}C_{F,t} - \overline{B}\right)
\end{aligned}$$

$$\begin{aligned}
\text{FOC}_H\text{:} \quad \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}} (1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{-1}{\eta}} &= \lambda P_{H,t} \\
\Rightarrow C_{H,t} &= (1-\alpha) P_{H,t}^{-\eta}\lambda^{-\eta}C_t \\
\text{FOC}_F\text{:} \quad \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}} \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{-1}{\eta}} &= \lambda P_{F,t} \\
\Rightarrow C_{F,t} &= \alpha P_{F,t}^{-\eta}\lambda^{-\eta}C_t
\end{aligned}$$

Dividing the two yields: $C_{F,t} = C_{H,t}\dfrac{\alpha}{1-\alpha}\left(\dfrac{P_{H,t}}{P_{F,t}}\right)^{\eta}$

Inserting into the budget constraint: $P_{H,t}C_{H,t} + P_{F,t}C_{H,t}\dfrac{\alpha}{1-\alpha}\left(\dfrac{P_{H,t}}{P_{F,t}}\right)^{\eta} = \overline{B}$

$$P_{H,t}C_{H,t}\left(1 + \frac{\alpha}{1-\alpha}\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\eta}\right) = \overline{B}$$

$$C_{H,t} = \frac{\overline{B}}{P_{H,t}}\left(1 + \frac{\alpha}{1-\alpha}\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\eta}\right)^{-1}$$

$$
\begin{aligned}
\text{CPI: } P_t^{1-\eta} &\equiv (1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \\
\Leftrightarrow \alpha P_{F,t}^{1-\eta} &= P_t^{1-\eta} - (1-\alpha) P_{H,t}^{1-\eta}
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow P_{H,t} C_{H,t} \left(1 + \frac{1}{1-\alpha} P_{H,t}^{\eta-1} \left(P_t^{1-\eta} - (1-\alpha) P_{H,t}^{1-\eta}\right)\right) &= \overline{B} \\
P_{H,t} C_{H,t} \left(1 + \frac{1}{1-\alpha} \left(\frac{P_t}{P_{H,t}}\right)^{1-\eta} - 1\right) &= \overline{B} \\
P_{H,t} C_{H,t} \frac{1}{1-\alpha} \left(\frac{P_t}{P_{H,t}}\right)^{1-\eta} &= \overline{B}
\end{aligned}
$$

$$
\begin{aligned}
&C_{H,t} = (1-\alpha) \frac{\overline{B}}{P_{H,t}} \left(\frac{P_{H,t}}{P_t}\right)^{1-\eta} \\
&C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} P_t^{-1} \overline{B} \\
&C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \\
&\text{Since } P_t C_t = \overline{B}
\end{aligned}
$$

$$
\begin{aligned}
\text{This also gives } C_{F,t} =& C_{H,t} \frac{\alpha}{1-\alpha} \left(\frac{P_{H,t}}{P_{F,t}}\right)^{\eta} \\
=& \frac{\alpha}{1-\alpha} (1-\alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \left(\frac{P_{H,t}}{P_{F,t}}\right)^{\eta} \\
=& \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t
\end{aligned}
$$

The price indices are:

$$
P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\varepsilon} di\right]^{\frac{1}{\varepsilon-1}}, \quad P_{F,t} = \left[\int_0^1 P_{F,t}(i)^{1-\varepsilon} di\right]^{\frac{1}{\varepsilon-1}}
$$

$$
\text{and CPI } P_t \equiv \left[(1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}},
$$

and the consumption indices are given by:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

$$\text{where } C_{H,t} = \left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\text{and } C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

We find demand for good j by maximization of consumption given a nominal budget constraint $\overline{M}$

$$\max_{C_{H,t}(s)} C_{H,t} = \left[\int_0^1 C_{H,t}(s)^{\frac{\varepsilon-1}{\varepsilon}} ds\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\text{s.t.} \int_0^1 P_{H,t}(s) C_{H,t}(s) ds = \overline{M}, \quad s = i, j$$

$$\mathcal{L} = \left[\int_0^1 C_{H,t}(s)^{\frac{\varepsilon-1}{\varepsilon}} ds\right]^{\frac{\varepsilon}{\varepsilon-1}} - \lambda\left(\int_0^1 P_{H,t}(s) C_{H,t}(s) ds - \overline{M}\right)$$

$$\text{FOC}_i\text{:} \left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{1}{\varepsilon-1}} C_{H,t}(i)^{\frac{-1}{\varepsilon}} = \lambda P_{H,t}(i)$$

$$C_{H,t} C_{H,t}(i)^{-1} = \lambda^{\varepsilon} P_{H,t}(i)^{\varepsilon}$$

$$C_{H,t}(i) = \lambda^{-\varepsilon} \frac{C_{H,t}}{P_{H,t}(i)^{\varepsilon}}$$

$$\text{For good j: } C_{H,t}(j) = \lambda^{-\varepsilon} \frac{C_{H,t}}{P_{H,t}(j)^{\varepsilon}}$$

$$\text{Dividing the two yields } C_{H,t}(i) = C_{H,t}(j) \left(\frac{P_{H,t}(j)}{P_{H,t}(i)}\right)^{\varepsilon}$$

$$
\begin{aligned}
\text{In budget } \int_0^1 P_{H,t}(i) C_{H,t}(j) \left(\frac{P_{H,t}(j)}{P_{H,t}(i)}\right)^{\varepsilon} di &= \overline{M} \\
C_{H,t}(j) P_{H,t}(j)^{\varepsilon} \int_0^1 P_{H,t}(i)^{1-\varepsilon} di &= \overline{M} \\
\text{Using } P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\varepsilon} di\right]^{\frac{1}{\varepsilon-1}} &\Leftrightarrow \int_0^1 P_{H,t}(i)^{1-\varepsilon} di = P_{H,t}^{\varepsilon-1} \\
\Rightarrow C_{H,t}(j) P_{H,t}(j)^{\varepsilon} P_{H,t}^{\varepsilon-1} &= \overline{M}
\end{aligned}
$$

$$
C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \frac{\overline{M}}{P_{H,t}} = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t},
$$

since $P_{H,t} C_{H,t} = \overline{M}$

B.2 Households

$$
\mathcal{L} = E_t \sum_{i=0}^{\infty} \beta^i \left\{ \begin{array}{c} \left[\frac{\left(C^j_{t+i} - hC_{t+i-1}\right)^{1-\sigma}}{1-\sigma} - \frac{\left(N^j_{t+i}\right)^{1+\varphi}}{1+\varphi} \right] \\ -\lambda_{1,t+i} \left[\begin{array}{c} C^j_{t+i} + \frac{B^j_{t+i}}{(1+r_{t+i})P_{t+i}} + \frac{S_{t+i} B^{f,j}_{t+i}}{(1+r^f_{t+i})\Phi(A_{t+i})P_{t+i}} \\ -\frac{B^j_{t+i-1}}{P_{t+i}} - \frac{S_{t+i} B^{f,j}_{t+i-1}}{P_{t+i}} - \frac{W_{t+i}}{P_{t+i}} N_{t+i} \end{array} \right] \end{array} \right\}.
$$

Maximizing with respect to C^j_{t+i}, B^j_{t+i}, $B^{f,j}_{t+i}$ and N^j_{t+i} we obtain the first order conditions

$$
wrt. C^j_{t+i} : E_t \left[\left(C^j_{t+i} - hC_{t+i-1}\right)^{-\sigma} - \lambda_{1,t+i} \right] = 0 \tag{B-1}
$$

$$
wrt. B^j_{t+i} : E_t \left[\lambda_{1,t+i+1} \beta^{i+1} \frac{1}{P_{t+i+1}} - \lambda_{1,t+i} \beta^i \frac{1}{(1+r_{t+i})P_{t+i}} \right] = 0 \tag{B-2}
$$

$$
wrt. B^{f,j}_{t+i} : E_t \left[\lambda_{1,t+i+1} \beta^{i+1} \frac{S_{t+i+1}}{P_{t+i+1}} - \lambda_{1,t+i} \beta^i \frac{S_{t+i}}{\left(1+r^f_{t+i}\right) \Phi\left(A_{t+i}\right) P_{t+i}} \right] = 0 \tag{B-3}
$$

$$
wrt. N^j_{t+i} : E_t \left[\lambda_{1,t+i} \frac{W_{t+i}}{P_{t+i}} - N^{\varphi}_{t+i} \right] = 0. \tag{B-4}
$$

From (B-1) and (B-2) we get

$$
E_t \left[\left(\frac{C^j_{t+1} - hC_t}{C^j_t - hC_{t-1}} \right)^{-\sigma} \right] = E_t \left[\frac{\lambda_{1,t+1}}{\lambda_{1,t}} \right] \text{ and } E_t \left[\frac{\lambda_{1,t+1}}{\lambda_{1,t}} \right] = E_t \left[\frac{P_{t+1}}{\beta(1+r_t)P_t} \right].
$$

Combined they yield the consumption Euler equation (19)

$$\beta(1+r_t)E_t\left[\left(\frac{C^j_{t+1}-hC_t}{C^j_t-hC_{t-1}}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right]=1.$$

Likewise (B-2) and (B-3) yield the condition for uncovered interest rate parity (20)

$$\begin{aligned}
E_t\left[\frac{S_tP_{t+1}}{\beta S_{t+1}\left(1+r^f_t\right)\Phi\left(A_t\right)P_t}\right] &= E_t\left[\frac{\lambda_{1,t+1}}{\lambda_{1,t}}\right]\\
E_t\left[\frac{S_tP_{t+1}}{\beta S_{t+1}\left(1+r^f_t\right)\Phi\left(A_t\right)P_t}\right] &= E_t\left[\frac{P_{t+1}}{\beta(1+r_t)P_t}\right]\\
\Leftrightarrow \frac{1+r_t}{1+r^f_t} &= E_t\left[\frac{S_{t+1}}{S_t}\right]\Phi\left(A_t\right).
\end{aligned}$$

From (B-1) and (B-4) we find the intratemporal optimality condition (21)

$$\frac{W_{t+i}}{P_{t+i}}=\frac{N^{\varphi}_{t+i}}{\left(C^j_{t+i}-hC_{t+i-1}\right)^{-\sigma}}.$$

B.3 Producers optimal price

$$\max_{P_{H,t}(i)} E_t\sum_{\tau=t}^{\infty}\left\{D_{t,\tau}\left[P_{H,\tau}(i)-MC_{H,\tau}(i)\right]C^T_{H,\tau}(i)\left[1-\Gamma_{PC_h,\tau}(i)\right]\right\},$$

The first two periods[34] can be written as

$$\begin{aligned}
&P_{H,t}(i)C^T_{H,t}(i)-P_{H,t}(i)C^T_{H,t}(i)\Gamma_{PC_H,t}(i)-MC_{H,t}(i)C^T_{H,t}(i)\\
&+MC_{H,t}(i)C^T_{H,t}(i)\Gamma_{PC_H,t}(i)+E_tD_{t,t+1}P_{H,t+1}(i)C^T_{H,t+1}(i)\\
&-E_tD_{t,t+1}P_{H,t+1}(i)C^T_{H,t+1}(i)\Gamma_{PC_H,t+1}(i)-E_tD_{t,t+1}MC_{H,t+1}(i)C^T_{H,t+1}(i)\\
&+E_tD_{t,t+1}MC_{H,t+1}(i)C^T_{H,t+1}(i)\Gamma_{PC_H,t+1}(i)
\end{aligned}$$

[34] These are the only periods that contain $P_{H,t}(i)$, thus the rest of the periods can be disregarded in the optimization.

Derivation with respect to $P_{H,t}(i)$ yields

$$
\begin{aligned}
&C^T_{H,t}(i) + P_{H,t}(i)\frac{\partial C^T_{H,t}(i)}{\partial P_{H,t}(i)} - C^T_{H,t}(i)\Gamma_{PC_H,t}(i) - P_{H,t}(i)\frac{\partial C^T_{H,t}(i)}{\partial P_{H,t}(i)}\Gamma_{PC_H,t}(i)\\
&- P_{H,t}(i)C^T_{H,t}(i)\frac{\partial \Gamma_{PC_H,t}(i)}{\partial P_{H,t}(i)} - MC_{H,t}(i)\frac{\partial C^T_{H,t}(i)}{\partial P_{H,t}(i)} + MC_{H,t}(i)\frac{\partial C^T_{H,t}(i)}{\partial P_{H,t}(i)}\Gamma_{PC_H,t}(i)\\
&+ MC_{H,t}(i)C^T_{H,t}(i)\frac{\partial \Gamma_{PC_H,t}(i)}{\partial P_{H,t}(i)} - E_t D_{t,t+1}P_{H,t+1}(i)C^T_{H,t+1}(i)\frac{\partial \Gamma_{PC_H,t+1}(i)}{\partial P_{H,t}(i)}\\
&+ E_t D_{t,t+1}MC_{H,t+1}(i)C^T_{H,t+1}(i)\frac{\partial \Gamma_{PC_H,t+1}(i)}{\partial P_{H,t}(i)}.
\end{aligned}
$$

Dividing by $C^T_{H,t}(i)$, multiplying by $P_{H,t}(i)$ and using that $P_{H,t}(i)\frac{\partial C^T_{H,t}(i)}{\partial P_{H,t}(i)} = -\varepsilon C^T_{H,t}(i)$, and finally collecting terms, we get

$$
\begin{aligned}
&[1-\Gamma_{PC_H,t}(i)]\,[P_{H,t}(i)\,(1-\varepsilon) + \varepsilon MC_{H,t}(i)]\\
&- [P_{H,t}(i) - MC_{H,t}(i)]\,P_{H,t}(i)\frac{\partial \Gamma_{PC_H,t}(i)}{\partial P_{H,t}(i)}\\
&- E_t D_{t,t+1}\frac{C^T_{H,t+1}(i)}{C^T_{H,t}(i)}\,[P_{H,t+1}(i) - MC_{H,t+1}(i)]\,P_{H,t}(i)\frac{\partial \Gamma_{PC_H,t+1}(i)}{\partial P_{H,t}(i)}.
\end{aligned}
$$

Since we have

$$
\begin{aligned}
\frac{\partial \Gamma_{PC_H,t}(i)}{\partial P_{H,t}(i)} &= \phi_{C_{H1}}\left(\frac{P_{H,t}(i)}{\pi P_{H,t-1}(i)} - 1\right)\frac{1}{\pi P_{H,t-1}(i)}\\
&\quad + \phi_{C_{H2}}\left(\frac{P_{H,t}(i)/P_{H,t-1}(i)}{P_{t-1}/P_{t-2}} - 1\right)\frac{1/P_{H,t-1}(i)}{P_{t-1}/P_{t-2}}\\
&\text{and}\\
\frac{\partial \Gamma_{PC_H,t+1}(i)}{\partial P_{H,t}(i)} &= \phi_{C_{H1}}\left(\frac{P_{H,t+1}(i)}{\pi P_{H,t}(i)} - 1\right)\left(-\frac{P_{H,t+1}(i)}{\pi P_{H,t}(i)^2}\right)\\
&\quad + \phi_{C_{H2}}\left(\frac{P_{H,t+1}(i)/P_{H,t}(i)}{P_t/P_{t-1}} - 1\right)\left(-\frac{P_{H,t+1}(i)/\left(P_{H,t}(i)\right)^2}{P_t/P_{t-1}}\right),
\end{aligned}
$$

we get the following first order condition for the optimal price

$$\begin{aligned}
0 &= [1-\Gamma_{PC_H,t}(i)]\,[P_{H,t}(i)\,(1-\varepsilon)+\varepsilon MC_{H,t}(i)] \\
&- [P_{H,t}(i)-MC_{H,t}(i)]\,\frac{\phi_{C_{H1}}P_{H,t}(i)}{\pi P_{H,t-1}(i)}\left(\frac{P_{H,t}(i)}{\pi P_{H,t-1}(i)}-1\right) \\
&- [P_{H,t}(i)-MC_{H,t}(i)]\,\frac{\phi_{C_{H2}}P_{H,t}(i)/P_{H,t-1}(i)}{P_{t-1}/P_{t-2}}\left(\frac{P_{H,t}(i)/P_{H,t-1}(i)}{P_{t-1}/P_{t-2}}-1\right) \\
&+ E_t D_{t,t+1}\frac{C^T_{H,t+1}(i)}{C^T_{H,t}(i)}\,[P_{H,t+1}(i)-MC_{H,t+1}(i)] \\
&\times\left[\begin{array}{c}\left(\frac{\phi_{C_{H1}}P_{H,t+1}(i)}{\pi P_{H,t}(i)}\right)\left(\frac{P_{H,t+1}(i)}{\pi P_{H,t}(i)}-1\right)\\ +\left(\frac{\phi_{C_{H2}}P_{H,t+1}(i)/P_{H,t}(i)}{P_t/P_{t-1}}\right)\left(\frac{P_{H,t+1}(i)/P_{H,t}(i)}{P_t/P_{t-1}}-1\right)\end{array}\right],
\end{aligned}$$

which, for the given marginal costs, are equations (8) and (9).

B.4 Calvo pricing

Firm i that is allowed to change its price will set price to minimize

$$\frac{1}{2}E_t\sum_{j=0}^{\infty}\beta^j\left(p^f_{i,t+j}-\widehat{mc}^n_{t+j}\right)^2,$$

where $\widehat{mc}^n$ is nominal marginal cost as percentage deviation from steady state. There is only probability $1-\theta$ that firm i will be allowed to change its price, so what will be depending on price at time t, will be

$$\left(p^f_{i,t}-\widehat{mc}^n_t\right)^2+\theta\beta E_t\left(p^f_{i,t}-\widehat{mc}^n_{t+1}\right)^2+\theta^2\beta^2E_t\left(p^f_{i,t}-\widehat{mc}^n_{t+2}\right)^2+\cdots$$

This gives first order condition

$$p^f_{i,t}\sum_{j=0}^{\infty}\theta^i\beta^i-\sum_{j=0}^{\infty}\theta^i\beta^iE\widehat{mc}^n_{t+j}=0$$

Since $\sum_{j=0}^{\infty}\theta^i\beta^i\approx 1/(1-\theta\beta)$, we get

$$p^f_{i,t}=(1-\theta\beta)\sum_{j=0}^{\infty}\theta^i\beta^iE\widehat{mc}^n_{t+j},$$

which is equation (14).

B.5 Equilibrium

$$\begin{aligned}
C_t + \frac{S_t B_t^f}{\left(1+r_t^f\right)\Phi(A_t)P_t} &= \frac{S_t B_{t-1}^f}{P_t} + \frac{W_t}{P_t}\frac{Y_t}{Z_t^Y} + \left(\frac{P_{H,t}}{P_t} - \frac{W_t}{P_t Z_t^Y}\right)\left(C_{H,t} + C_{H,t}^f\right) \\
&\quad + \left(\frac{P_{F,t}}{P_t} - \frac{S_t P_{F,t}^*}{P_t}\right) C_{F,t} \\
C_t + \frac{S_t B_t^f}{\left(1+r_t^f\right)\Phi(A_t)P_t} &= \frac{S_t B_{t-1}^f}{P_t} + \left(\frac{P_{H,t}}{P_t} - \frac{W_t}{P_t Z_t^Y} + \frac{W_t}{P_t Z_t^Y}\right)\left(C_{H,t} + C_{H,t}^f\right) \\
&\quad + \left(\frac{P_{F,t}}{P_t} - \frac{S_t P_{F,t}^*}{P_t}\right) C_{F,t} \\
P_t C_t + \frac{S_t B_t^f}{\left(1+r_t^f\right)\Phi(A_t)} &= S_t B_{t-1}^f + P_{H,t}C_{H,t} + P_{H,t}C_{H,t}^f + P_{F,t}C_{F,t} - S_t P_{F,t}^* C_{F,t} \\
\frac{S_t B_t^f}{\left(1+r_t^f\right)\Phi(A_t)} - S_t B_{t-1}^f &= P_{H,t}C_{H,t}^f - S_t P_{F,t}^* C_{F,t},
\end{aligned}$$

which is equation (25)

B.6 Steady state

In steady state inflation is zero, so we have $P_t = P_{t+1} = P$. Consumption is at a constant level, so $C_t = C_{t+1} = C$. From the consumption Euler equation (19) we then get

$$\beta(1+r_t)E_t\left[\left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right] = 1$$
$$\Rightarrow \text{In steady state: } \beta(1+r) = 1$$

If we assume that foreign consumers face a similar maximization problem, and that they have the same discount factor, β, we will get $r = r^f$ in steady state. From the first order condition with respect to bond holdings (B-2), we also see that in steady state, $\lambda_t = \lambda_{t+1} = \lambda$. Then the first order condition on foreign bond holdings (B-3) yields

$$\begin{aligned}
\beta &= \frac{1}{\left(1+r^f\right)\Phi(A)} \\
\Phi(A) &= \frac{1+r}{1+r^f} \\
\Phi(A) &= 1.
\end{aligned}$$

And since

$$\Phi(A) = e^{-\phi A} = 1,$$

we have

$$A \equiv SB^f/P = 0,$$

which says that in steady state, net foreign bond holdings are zero. Aggregating the budget constraint (18) under the assumption that domestic bonds are zero in net supply, we see that in steady state, consumption is equal to the sum of real wage income and real profits

$$C = \frac{W}{P}N + X.$$

If we normalize the terms of trade P_F/P_H to unity, we will have

$$P \equiv \left[(1-\alpha)\, P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} = P_F = P_H,$$

and domestic demand for the two types of goods will then be

$$C_H = (1-\alpha)\left(\frac{P_H}{P}\right)^{-\eta} C = (1-\alpha)\, C$$

$$\text{and } C_F = \alpha \left(\frac{P_F}{P}\right)^{-\eta} C = \alpha C.$$

C Log-linearizing

I will use both Taylor approximation and the short cuts as described in Uhlig (1999). The first order Taylor approximation of $f(x_t, y_t)$ around its steady state $f(x, y)$ is $f(x_t, y_t) \approx f(x, y) + f_x(x, y)(x_t - x) + f_y(x, y)(y_t - y)$. Now, if $\widehat{x}_t$ is percentage deviation in variable x_t from its steady state x, we have $\widehat{x}_t = \ln x_t - \ln x$ and $x_t = x \exp(\widehat{x}_t)$. When $\widehat{x}_t$ is small, $\widehat{x}_t \approx \ln(1 + \widehat{x}_t)$, so $\exp(\widehat{x}_t) \approx 1 + \widehat{x}_t$, and then $x_t \approx x\widehat{x}_t$ up to a constant.

C.1 Euler equation

$$\begin{aligned}
\beta(1+r_t)E_t\left[\left(\frac{C_{t+1}-hC_t}{C_t-hC_{t-1}}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right] &= 1\\
\exp(\widehat{r}_t)E_t\left[\left(\frac{\exp\widehat{c}_{t+1}-h\exp\widehat{c}_t}{\exp\widehat{c}_t-h\exp\widehat{c}_{t-1}}\right)^{-\sigma}\exp(\widehat{p}_t-\widehat{p}_{t+1})\right] &= 1\\
\exp(\widehat{r}_t)^{-\frac{1}{\sigma}}E_t\left[\left(\frac{\exp\widehat{c}_{t+1}-h\exp\widehat{c}_t}{\exp\widehat{c}_t-h\exp\widehat{c}_{t-1}}\right)\exp(\widehat{p}_t-\widehat{p}_{t+1})^{-\frac{1}{\sigma}}\right] &= 1\\
\exp(\widehat{r}_t)^{-\frac{1}{\sigma}}E_t\left[(\exp\widehat{c}_{t+1}-h\exp\widehat{c}_t)\exp(\widehat{p}_t-\widehat{p}_{t+1})^{-\frac{1}{\sigma}}\right] &= \exp\widehat{c}_t-h\exp\widehat{c}_{t-1}
\end{aligned}$$

$$1+\widehat{c}_t-h(1+\widehat{c}_{t-1}) = E_t\left[\begin{array}{c}(1+\widehat{c}_{t+1}-\frac{1}{\sigma}\widehat{p}_t+\frac{1}{\sigma}\widehat{p}_{t+1}-\frac{1}{\sigma}\widehat{r}_t)\\ -h(1+\widehat{c}_t-\frac{1}{\sigma}\widehat{p}_t+\frac{1}{\sigma}\widehat{p}_{t+1}-\frac{1}{\sigma}\widehat{r}_t)\end{array}\right]$$

$$\begin{aligned}
\Leftrightarrow \widehat{c}_t-h\widehat{c}_{t-1} =& E_t\widehat{c}_{t+1}-\frac{1}{\sigma}\widehat{p}_t+\frac{1}{\sigma}E_t\widehat{p}_{t+1}-\frac{1}{\sigma}\widehat{r}_t\\
&-h\widehat{c}_t+h\frac{1}{\sigma}\widehat{p}_t-h\frac{1}{\sigma}E_t\widehat{p}_{t+1}+h\frac{1}{\sigma}\widehat{r}_t\\
\Leftrightarrow \widehat{c}_t =& \frac{h}{(1+h)}\widehat{c}_{t-1}+\frac{1}{(1+h)}E_t\widehat{c}_{t+1}+\frac{(1-h)}{(1+h)}\frac{1}{\sigma}E_t\widehat{\pi}_{t+1}-\frac{(1-h)}{1+h}\frac{1}{\sigma}\widehat{r}_t\\
\Leftrightarrow \widehat{c}_t =& \frac{h}{(1+h)}\widehat{c}_{t-1}+\frac{1}{(1+h)}E_t\widehat{c}_{t+1}-\frac{(1-h)}{(1+h)}\frac{1}{\sigma}(\widehat{r}_t-E_t\widehat{\pi}_{t+1})
\end{aligned}$$

$$\text{where } E_t\widehat{\pi}_{t+1} = E_t\widehat{p}_{t+1}-\widehat{p}_t \text{ and } \widehat{r}_t = r_t - r \approx \ln\left[\frac{1+r_t}{1+r}\right]$$

C.2 Demand

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

$$C_t \approx C + \frac{\eta}{\eta-1}\left[(1-\alpha)^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}} \frac{\eta-1}{\eta}(1-\alpha)^{\frac{1}{\eta}} C_H^{\frac{-1}{\eta}}(C_{H,t}-C_H)$$
$$+ \frac{\eta}{\eta-1}\left[(1-\alpha)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}} \frac{\eta-1}{\eta}\alpha^{\frac{1}{\eta}} C_F^{\frac{-1}{\eta}}(C_{F,t}-C_F)$$

$$C_t \approx C + C^{\frac{1}{\eta}}(1-\alpha)^{\frac{1}{\eta}} C_H^{\frac{-1}{\eta}}(C_{H,t}-C_H)$$
$$+ C^{\frac{1}{\eta}}\alpha^{\frac{1}{\eta}} C_F^{\frac{-1}{\eta}}(C_{F,t}-C_F)$$

$$C_t \approx C + C^{\frac{1}{\eta}}(1-\alpha)^{\frac{1}{\eta}} C_H^{\frac{-1}{\eta}} C_{H,t} - C^{\frac{1}{\eta}}\left((1-\alpha)^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}}\right)$$
$$+ C^{\frac{1}{\eta}}\alpha^{\frac{1}{\eta}} C_F^{\frac{-1}{\eta}} C_{F,t}$$

$$C_t \approx C + C^{\frac{1}{\eta}}(1-\alpha)^{\frac{1}{\eta}} C_H^{\frac{-1}{\eta}} C_{H,t} - C^{\frac{1}{\eta}} C^{\frac{\eta-1}{\eta}}$$
$$+ C^{\frac{1}{\eta}}\alpha^{\frac{1}{\eta}} C_F^{\frac{-1}{\eta}} C_{F,t}$$

$$C_t \approx C^{\frac{1}{\eta}}\left((1-\alpha)^{\frac{1}{\eta}} C_H^{\frac{-1}{\eta}} C_{H,t} + \alpha^{\frac{1}{\eta}} C_F^{\frac{-1}{\eta}} C_{F,t}\right)$$
$$C^{\frac{-1}{\eta}} C_t \approx \left((1-\alpha)^{\frac{1}{\eta}} C_H^{\frac{-1}{\eta}} C_{H,t} + \alpha^{\frac{1}{\eta}} C_F^{\frac{-1}{\eta}} C_{F,t}\right)$$

$$C^{\frac{\eta-1}{\eta}} \exp(\widehat{C}_t) \approx (1-\alpha)^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} \exp(\widehat{C}_{H,t}) + \alpha^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}} \exp(\widehat{C}_{F,t})$$
$$C^{\frac{\eta-1}{\eta}}(1+\widehat{C}_t) \approx (1-\alpha)^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}}(1+\widehat{C}_{H,t}) + \alpha^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}}(1+\widehat{C}_{F,t})$$
$$C^{\frac{\eta-1}{\eta}}\widehat{C}_t \approx (1-\alpha)^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}}\widehat{C}_{H,t} + \alpha^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}}\widehat{C}_{F,t}$$

$$\widehat{C}_t \approx \frac{(1-\alpha)^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}}\widehat{C}_{H,t} + \alpha^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}}\widehat{C}_{F,t}}{\left((1-\alpha)^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}}\right)} = (1-\gamma c)\,\widehat{C}_{H,t} + \gamma_c\widehat{C}_{F,t},$$

where γ_c is import share of consumption. This is equation (34).

$$
\begin{aligned}
C_{H,t} &= (1-\alpha)\left(p_{H,t}\right)^{-\eta} C_t \\
C_H \exp(\widehat{C}_{H,t}) &= (1-\alpha)\left(p_H \exp(\widehat{p}_{H,t})\right)^{-\eta} C \exp(\widehat{C}_t) \\
C_H(1+\widehat{C}_{H,t}) &= (1-\alpha)\left((1-\eta\widehat{p}_{H,t}+\widehat{C}_t)\right) C \\
1+\widehat{C}_{H,t} &= 1-\eta\widehat{p}_{H,t}+\widehat{C}_t \\
\widehat{C}_{H,t} &= \widehat{C}_t-\eta\widehat{p}_{H,t}
\end{aligned}
$$

Likewise

$$
\widehat{C}_{F,t} = \widehat{C}_t-\eta\widehat{p}_{F,t}
$$

and

$$
\begin{aligned}
C^f_{H,t} &= \alpha^f\left(\frac{P_{H,t}}{S_t P^f_t}\right)^{-\eta} C^f_t \\
C^f_{H,t} &= \alpha^f\left(\frac{p_{H,t}}{Q_t}\right)^{-\eta} C^f_t \\
C^f_H \exp\left(\widehat{C}^f_{H,t}\right) &= \alpha^f\left(\frac{p_H}{Q}\exp\left(\widehat{p}_{H,t}-\widehat{Q}_t\right)\right)^{-\eta} C^f \exp\left(\widehat{C}^f_t\right) \\
C^f_H\left(1+\widehat{C}^f_{H,t}\right) &= \alpha^f\left(\frac{p_H}{Q}\right)^{-\eta}\left(1-\eta\left(\widehat{p}_{H,t}-\widehat{Q}_t\right)+\widehat{C}^f_t\right) C^f \\
\widehat{C}^f_{H,t} &= \widehat{C}^f_t-\eta\left(\widehat{p}_{H,t}-\widehat{Q}_t\right)
\end{aligned}
$$

We then have the following aggregate demand for domestic produced goods

$$
\begin{aligned}
C^T_{H,t} &= C_{H,t}+C^f_{H,t}+G_t \\
C^T_H + C^T_H\widehat{C}^T_{H,t} &= C_H + C_H\widehat{C}_{H,t} + C^f_H + C^f_H\widehat{C}^f_{H,t} + G + G\widehat{G}_t \\
C^T_H\widehat{C}^T_{H,t} &= C_H\widehat{C}_{H,t} + C^f_H\widehat{C}^f_{H,t} + G\widehat{G}_t \\
\widehat{C}^T_{H,t} &= \frac{C_H}{C^T_H}\widehat{C}_{H,t} + \frac{C^f_H}{C^T_H}\widehat{C}^f_{H,t} + \frac{G}{C^T_H}\widehat{G}_t
\end{aligned}
$$

And in equilibrium

$$
\widehat{Y}_t = \frac{C_H}{C^T_H}\widehat{C}_{H,t} + \frac{C^f_H}{C^T_H}\widehat{C}^f_{H,t} + \frac{G}{C^T_H}\widehat{G}_t,
$$

which is equation (32).

C.3 UIP

$$
\begin{aligned}
\frac{1+r_t}{1+r_t^f} &= E_t\left[\frac{S_{t+1}}{S_t}\right]\Phi\left(A_t\right)\\
\frac{1+r_t}{1+r_t^f} &= E_t\left[\frac{Q_{t+1}}{Q_t}\frac{\pi_{t+1}}{\pi_{t+1}^f}\right]\Phi\left(A_t\right)\\
\frac{R}{R^f}\exp\left(\widehat{R}_t-\widehat{R}_t^f\right) &= E_t\left[\exp\left(\widehat{Q}_{t+1}-\widehat{Q}_t+\widehat{\pi}_{t+1}-\widehat{\pi}_{t+1}^f-\phi A_t+Z_t^B\right)\right]\\
\left(1+\widehat{R}_t-\widehat{R}_t^f\right) &= \left(1+E_t\widehat{Q}_{t+1}-\widehat{Q}_t+E_t\widehat{\pi}_{t+1}-E_t\widehat{\pi}_{t+1}^f-\phi A_t+Z_t^B\right)\\
\widehat{R}_t-\widehat{R}_t^f &= E_t\widehat{Q}_{t+1}-\widehat{Q}_t+E_t\widehat{\pi}_{t+1}-E_t\widehat{\pi}_{t+1}^f-\phi A_t+Z_t^B,
\end{aligned}
$$

which is equation (35).

C.4 Risk sharing

Aggregated budget constraint (22) when $b_t^f=\frac{B_t^f}{P_t^f}$, $Q_t=\frac{S_tP_t^f}{P_t}$, $\Phi(A_t)=e^{-\phi A_t+Z_t^B}$, $A_t=\frac{S_tB_t^f}{P_t}=Q_tb_t^f$, $A=0$, $1+r^f=\frac{1}{\beta}$, and $C=\frac{W}{P}N+X$

$$
\begin{aligned}
\frac{S_tB_t^f}{\left(1+r_t^f\right)\Phi(A_t)}-S_tB_{t-1}^f &= P_{H,t}C_{H,t}^f-S_tP_{F,t}^fC_{F,t}\\
\frac{Q_tb_t^f}{\left(1+r_t^f\right)\Phi(Q_tb_t^f)}-\frac{Q_tB_{t-1}^f}{P_t^f} &= p_{H,t}C_{H,t}^f-Q_tp_{F,t}^fC_{F,t}\\
\frac{Q_tb_t^f}{\left(1+r_t^f\right)\Phi(Q_tb_t^f)}-\frac{Q_tb_{t-1}^f}{\pi_t^f} &= p_{H,t}C_{H,t}^f-Q_tp_{F,t}^fC_{F,t}\\
&\Rightarrow p_HC_H^f=Q_tC_F \quad \text{in steady state}
\end{aligned}
$$

$$
\begin{aligned}
\frac{Q\widehat{b}_t^f}{1+r^f}-\frac{Q\widehat{b}_{t-1}^f}{\pi^f} &= p_HC_H^f\exp\left(\widehat{p}_{H,t}+\widehat{C}_{H,t}^f\right)-QC_F\exp\left(\widehat{Q}_t+\widehat{C}_{F,t}\right)\\
\frac{Q\widehat{b}_t^f}{1+r^f}-\frac{Q\widehat{b}_{t-1}^f}{\pi^f} &= p_HC_H^f\left(\widehat{p}_{H,t}+\widehat{C}_{H,t}^f\right)-QC_F\left(\widehat{Q}_t+\widehat{C}_{F,t}\right),
\end{aligned}
$$

which is equation (36).

C.5 Intratemporal optimality condition

$$
\begin{aligned}
w_t &= \frac{N_t^{\varphi}}{(C_t - hC_{t-1})^{-\sigma}} \\
w_t^{\frac{1}{\sigma}} N_t^{-\frac{\varphi}{\sigma}} &= C_t - hC_{t-1} \\
w^{\frac{1}{\sigma}} N^{-\frac{\varphi}{\sigma}} \exp(\frac{1}{\sigma}\widehat{w}_t - \frac{\varphi}{\sigma}\widehat{N}_t) &= C\exp(\widehat{C}_t) - hC\exp(\widehat{C}_{t-1}) \\
w^{\frac{1}{\sigma}} N^{-\frac{\varphi}{\sigma}} (1 + \frac{1}{\sigma}\widehat{w}_t - \frac{\varphi}{\sigma}\widehat{N}_t) &= C(1 + \widehat{C}_t) - hC(1 + \widehat{C}_{t-1})
\end{aligned}
$$

$$
\text{In SS: } w^{\frac{1}{\sigma}} N^{-\frac{\varphi}{\sigma}} = (1-h)C
$$

$$
\begin{aligned}
w^{\frac{1}{\sigma}} N^{-\frac{\varphi}{\sigma}} (\frac{1}{\sigma}\widehat{w}_t - \frac{\varphi}{\sigma}\widehat{N}_t) &= C\widehat{C}_t - hC\widehat{C}_{t-1} \\
\frac{1-h}{\sigma}\widehat{w}_t - (1-h)\frac{\varphi}{\sigma}\widehat{N}_t &= \widehat{C}_t - h\widehat{C}_{t-1} \\
\frac{1}{\varphi}\widehat{w}_t - \frac{\sigma}{\varphi(1-h)}\widehat{C}_t + \frac{\sigma h}{\varphi(1-h)}\widehat{C}_{t-1} &= \widehat{N}_t,
\end{aligned}
$$

which is equation (37).

C.6 Producers' optimal price

$$
\begin{aligned}
0 = {} & [1 - \Gamma_{PCt}] \left[p_{H,t}(1-\varepsilon_t) + \varepsilon_t mc_t\right] \\
& - [p_{H,t} - mc_t] \frac{\phi_{CH1}\pi_t^H}{\pi}\left(\frac{\pi_t^H}{\pi} - 1\right) \\
& - [p_{H,t} - mc_t] \frac{\phi_{CH2}\pi_t^H}{\pi_{t-1}^H}\left(\frac{\pi_t^H}{\pi_{t-1}^H} - 1\right) \\
& + E_t D_{t,t+1}\pi_{t+1}\frac{C_{H,t+1}^T}{C_{H,t}^T}[p_{H,t+1} - mc_{t+1}] \\
& \times \left[\begin{array}{c} \left(\frac{\phi_{CH1}\pi_{t+1}^H}{\pi}\right)\left(\frac{\pi_{t+1}^H}{\pi} - 1\right) \\ + \left(\frac{\phi_{CH2}\pi_{t+1}^H}{\pi_t^H}\right)\left(\frac{\pi_{t+1}^H}{\pi_t^H} - 1\right) \end{array}\right].
\end{aligned}
$$

Using that $f(x_t, y_t) \approx f(x, y) + f'_x(x, y)(x_t - x) + f'_y(x, y)(y_t - y) = f'_y(x, y)y_t$ when $y = 0$ and $f(x_t, y_t) = x_t y_t$

$$
\begin{aligned}
0 &= p_{H,t}(1-\varepsilon_t) + \varepsilon_t mc_t \\
&- [p_H - mc] \frac{\phi_{CH1}\pi_t^H}{\pi}\left(\frac{\pi_t^H}{\pi} - 1\right) \\
&- [p_H - mc] \frac{\phi_{CH2}\pi_t^H}{\pi_{t-1}^H}\left(\frac{\pi_t^H}{\pi_{t-1}^H} - 1\right) \\
&+ E_t D\pi \frac{C_H^T}{C_H^T}[p_H - mc] \\
&\times \left[\begin{array}{c} \left(\frac{\phi_{CH1}\pi_{t+1}^H}{\pi}\right)\left(\frac{\pi_{t+1}^H}{\pi} - 1\right) \\ + \left(\frac{\phi_{CH2}\pi_{t+1}^H}{\pi_t^H}\right)\left(\frac{\pi_{t+1}^H}{\pi_t^H} - 1\right) \end{array}\right].
\end{aligned}
$$

SS

$$
\begin{aligned}
D &= \beta \\
\pi &= 1
\end{aligned}
$$

$$
\begin{aligned}
0 &= p_H(1-\varepsilon) + \varepsilon mc \\
\Rightarrow p_H &= \frac{\varepsilon}{\varepsilon - 1} mc, \\
p_H - mc &= \frac{1}{\varepsilon - 1} mc \\
\text{and } \frac{p_H}{mc} &= \frac{\varepsilon}{\varepsilon - 1}
\end{aligned}
$$

$$\Rightarrow$$

$$
\begin{aligned}
0 &= p_{H,t}(1-\varepsilon_t) + \varepsilon_t mc_t \\
&- \frac{1}{\varepsilon - 1} mc \frac{\phi_{CH1}\pi_t^H}{\pi}\left(\frac{\pi_t^H}{\pi} - 1\right) \\
&- \frac{1}{\varepsilon - 1} mc \frac{\phi_{CH2}\pi_t^H}{\pi_{t-1}^H}\left(\frac{\pi_t^H}{\pi_{t-1}^H} - 1\right) \\
&+ E_t \beta \frac{1}{\varepsilon - 1} mc \\
&\times \left[\begin{array}{c} \left(\frac{\phi_{CH1}\pi_{t+1}^H}{\pi}\right)\left(\frac{\pi_{t+1}^H}{\pi} - 1\right) \\ + \left(\frac{\phi_{CH2}\pi_{t+1}^H}{\pi_t^H}\right)\left(\frac{\pi_{t+1}^H}{\pi_t^H} - 1\right) \end{array}\right].
\end{aligned}
$$

$$
\begin{aligned}
0 &= p_H \exp(\widehat{p}_{H,t}) - \varepsilon p_H \exp(\widehat{p}_{H,t} + \widehat{\varepsilon}_t) + \varepsilon mc \exp(\widehat{mc}_{H,t} + \widehat{\varepsilon}_t) \\
&- \frac{1}{\varepsilon - 1} mc \phi_{CH1} \exp(\widehat{\pi}^H_t) \left(\exp(\widehat{\pi}^H_t) - 1\right) \\
&- \frac{1}{\varepsilon - 1} mc \phi_{CH2} \exp(\widehat{\pi}^H_t - \widehat{\pi}^H_{t-1}) \left(\exp(\widehat{\pi}^H_t - \widehat{\pi}^H_{t-1}) - 1\right) \\
&+ E_t \beta \frac{1}{\varepsilon - 1} mc \times \left[\phi_{CH1} \exp(\widehat{\pi}^H_{t+1}) \left(\exp(\widehat{\pi}^H_{t+1}) - 1\right) + \phi_{CH2} \exp(\widehat{\pi}^H_{t+1} - \widehat{\pi}^H_t) \left(\exp(\widehat{\pi}^H_{t+1} - \widehat{\pi}^H_t) - 1\right)\right]
\end{aligned}
$$

$\exp(\widehat{x}_t) \approx 1 + \widehat{x}_t$. Subtracting SS $0 = p_H (1 - \varepsilon) + \varepsilon mc$.Dividing by mc, and using that $\frac{p_H}{mc} = \frac{\varepsilon}{\varepsilon - 1}$. Using that $\widehat{x}_t \widehat{y}_t \approx 0$ and negligible. Solving for $\widehat{\pi}^H_t$, using that $\frac{1-x}{x-1} = -1$, and multiplying with $\varepsilon - 1$:

$$
\begin{aligned}
\widehat{\pi}^H_t = &- \frac{\varepsilon}{(\phi_{CH1} + (1 + \beta)\phi_{CH2})} \widehat{\varepsilon}_t + \frac{\varepsilon(\varepsilon - 1)}{(\phi_{CH1} + (1 + \beta)\phi_{CH2})} (\widehat{mc}_{H,t} - \widehat{p}_{H,t}) \\
&+ \frac{\phi_{CH2}}{(\phi_{CH1} + (1 + \beta)\phi_{CH2})} \widehat{\pi}^H_{t-1} + E_t \beta \frac{(\phi_{CH1} + \phi_{CH2})}{(\phi_{CH1} + (1 + \beta)\phi_{CH2})} \widehat{\pi}^H_{t+1}
\end{aligned}
$$

When $mc_{H,t} = \frac{W_t}{Z^Y_t P_t} = \frac{w_t}{Z^Y_t}$ and $mc_{F,t} = \frac{S_t P^f_{F,t}}{P_t} = Q_t$, we get

$$
\begin{aligned}
mc_H \exp(\widehat{mc}_{H,t}) &= \frac{w}{Z^Y} \exp\left(\widehat{w}_t - \widehat{Z}^Y_t\right) \\
1 + \widehat{mc}_{H,t} &= 1 + \widehat{w}_t - \widehat{Z}^Y_t \\
\widehat{mc}_{H,t} &= \widehat{w}_t - \widehat{Z}^Y_t
\end{aligned}
$$

and

$$
\widehat{mc}_{F,t} = \widehat{Q}_t
$$

And thus for imported inflation

$$
\begin{aligned}
\widehat{\pi}^F_t = &- \frac{\varepsilon}{(\phi_{CF1} + (1 + \beta)\phi_{CF2})} \widehat{\varepsilon}_t + \frac{\varepsilon(\varepsilon - 1)}{(\phi_{CF1} + (1 + \beta)\phi_{CF2})} \left(\widehat{Q}_t - \widehat{p}_{F,t}\right) \\
&+ \frac{\phi_{CF2}}{(\phi_{CF1} + (1 + \beta)\phi_{CF2})} \widehat{\pi}^F_{t-1} + E_t \beta \frac{(\phi_{CF1} + \phi_{CF2})}{(\phi_{CF1} + (1 + \beta)\phi_{CF2})} \widehat{\pi}^F_{t+1},
\end{aligned}
$$

which are equations (38) and (39)

D Dynare code for benchmark model

```
//---------------------------------------------------//
// Declaration of endogenous and exogenous variables  //
//---------------------------------------------------//

var y C CH CF CH_f C_f r rf bf z_y z_u z_r z_b pi pih pif pif_f ph pf w Q
N vepsHhat vepsFhat G dQSA_PCPIJAEI dQSA_PCPIJAEIMP logQUA_QI44 dQSA_YMN
QUA_RN3M dAUA_WILMN_PCT_Qr;
varexo xi_u xi_y xi_C_f xi_r xi_rf xi_b xi_pif_f xi_vepsH xi_vepsF xi_G;

//---------------------------------------------------//
// Declaration of parameters                          //
//---------------------------------------------------//

parameters alpha beta eta h gammac gammay omega_pi omega_y omega_r phi
phi_cf1  phi_cf2 phi_ch1  phi_ch2 sigma vepsilon vphi rho_u rho_r rho_rf
rho_y rho_b rho_C_f rho_pif_f rho_vepsH rho_vepsF rho_G GSS QSS phSS pfSS
CFSS CHSS CHTSS CH_fSS pi_fSS rSS ySS;

alpha = 0.32;
beta  = 0.993;
sigma   = 1;
vphi = 3; //2.5;
eta = 1.1;
chi = 2;
vepsilon= 6;
omega_pi = 1.5;
omega_y = 0.5;
omega_r = 0.7;
phi = 0.0002;
h = 0.75;
phi_ch1 = 1;
phi_ch2 = 1;
phi_cf1 = 1;
```

```
phi_cf2 = 1;
rho_u = 0.5;
rho_y = 0.5;
rho_b = 0.5;
rho_G = 0.5;
rho_r = 0;
rho_rf = 0.5;
rho_vepsH = 0.5;
rho_vepsF = 0.5;
rho_pif_f = 0.5;
rho_C_f = 0.5;

//SS values Dynare v.4
gammac = 0.32469;        //Import share of consumption
gammay = 0.12001;        //Export share of production
QSS = 0.72043;
phSS = 1.0717;
pfSS = 0.86452;
CHSS = 0.51597;
CFSS = 0.30754;
CH_fSS = 0.20674;
CHTSS = 1.7227;
GSS = 1;
pi_fSS = 1;
rSS = 1/beta;
ySS = 1.7227;


//---------------------------------------------------//
// DSGE model specification                          //
//---------------------------------------------------//

model(linear);

//Demand
C = (1-gammac)*CH+gammac*CF;
CH = C-eta*(ph);
```

```
CF = C-eta*(pf);
CH_f = C_f-eta*(ph-Q);
y = (CHSS/CHTSS)*CH+(CH_fSS/CHTSS)*CH_f+(GSS/CHTSS)*G;
y = z_y+N;

//Euler
r = (sigma/(1-h))*C(+1)-((1+h)/(1-h))*sigma*C+(h*sigma/(1-h))*C(-1)
+pi(+1);

//Intratemporal
w = vphi*N+(sigma/(1-h))*C-((sigma*h)/(1-h))*C(-1);

//Producer FOCs
pih =((vepsilon*(vepsilon-1))/(1000*phi_ch1+(1+beta)*1000*phi_ch2))*(w-z_y-ph)
+(1000*phi_ch2/(1000*phi_ch1+(1+beta)*1000*phi_ch2))*pih(-1)
+beta*((1000*phi_ch1+1000*phi_ch2)/(1000*phi_ch1+(1+beta)*1000*phi_ch2))*pih(+1)
-(vepsilon/(1000*phi_ch1+(1+beta)*1000*phi_ch2))*vepsHhat;

pif =((vepsilon*(vepsilon-1))/(1000*phi_cf1+(1+beta)*1000*phi_cf2))*(Q-pf)
+(1000*phi_cf2/(1000*phi_cf1+(1+beta)*1000*phi_cf2))*pif(-1)
+beta*((1000*phi_cf1+1000*phi_cf2)/(1000*phi_cf1+(1+beta)*1000*phi_cf2))*pif(+1)
-(vepsilon/(1000*phi_cf1+(1+beta)*1000*phi_cf2))*vepsFhat;

//UIP
r -rf= Q(+1)-Q+pi(+1)-pif_f(+1)-phi*QSS*bf+z_b;

//Taylor
r = omega_r*r(-1)+((1-omega_r)/rSS)*(omega_pi*pi+omega_y*ySS*(y-y(-1)))+xi_r;

//Bonds
beta*QSS*bf-QSS*bf(-1)/pi_fSS = phSS*CH_fSS*(ph+CH_f)-QSS*CFSS*(Q+CF);

//Pi
//pi = (1-alpha)*phSS^(1-eta)*pih+alpha*pfSS^(1-eta)*pif;
pif = pf-pf(-1)+pi;
pih = ph-ph(-1)+pi;
```

```
//AR1-processes
G = rho_G*G(-1)+xi_G;
vepsHhat = rho_vepsH*vepsHhat(-1)+xi_vepsH;
vepsFhat = rho_vepsF*vepsFhat(-1)+xi_vepsF;
pif_f = rho_pif_f*pif_f(-1)+xi_pif_f;
C_f = rho_C_f*C_f(-1)+xi_C_f;
rf = rho_rf*rf(-1)+xi_rf;
z_u = rho_u*z_u(-1)-xi_u;
z_y = rho_y*z_y(-1)+xi_y;
z_b = rho_b*z_b(-1)+xi_b;
z_r = rho_r*z_r(-1)+xi_r;

//Observables
dQSA_PCPIJAEI -1= pih;
dQSA_PCPIJAEIMP-1=pif;
logQUA_QI44=Q;
dQSA_YMN=y-y(-1);
QUA_RN3M=r;
dAUA_WILMN_PCT_Qr=w-w(-1);
end;

// Declaring observables
varobs dQSA_PCPIJAEI dQSA_PCPIJAEIMP logQUA_QI44 dQSA_YMN QUA_RN3M
dAUA_WILMN_PCT_Qr;

// Compute steady state
steady; //(solve_algo = 0);

// Compute eigenvalues and check Blanchard-Kahn conditions
check;

estimated_params;
rho_y, beta_pdf,  0.5, 0.2;
rho_b, beta_pdf,  0.5, 0.2;
rho_G, beta_pdf,  0.5, 0.2;
rho_vepsH, beta_pdf,  0.5, 0.2;
rho_vepsF, beta_pdf,  0.5, 0.2;
```

```
phi_ch1, INV_GAMMA_PDF, 0.15, inf;
phi_ch2, INV_GAMMA_PDF, 0.075, inf;
phi_cf1, INV_GAMMA_PDF, 0.15, inf;
phi_cf2, INV_GAMMA_PDF, 0.75, inf;

stderr xi_y,INV_GAMMA_PDF,0.02,inf;
stderr xi_b,INV_GAMMA_PDF,0.01,inf;
stderr xi_G,INV_GAMMA_PDF,0.012,inf;
stderr xi_r,INV_GAMMA_PDF,.0025,inf;
stderr xi_vepsH,INV_GAMMA_PDF,0.05,inf;
stderr xi_vepsF,INV_GAMMA_PDF,0.05,inf;
end;

estimation(datafile=dataest,prefilter=1,lik_init=1,mh_replic=1500000,
mh_jscale=0.5,mode_check);
```

E Definition of variables and parameters

Table 8: Variable descriptions

Var	Code	Description	Var	Code	Description
C_t	C	Total domestic dem.	r_t^f	rf	Foreign interest rate
$C_{H,t}$	CH	Dom. dem. dom. goods	N_t	N	Supply of labour
$C_{F,t}$	CF	Dom. dem. imp. goods	w_t	w	Real wage
$C_{H,t}^f$	CH_f	For. dem. of dom. goods	X_t		Real profits
C_t^f	C_f	Total foreign dem.	Y_t	y	Domestic production
P_t		Consumer price index	Z_t^Y	z_y	Tot. factor pr. in prod.
$P_{H,t}$		Price on domestic goods	Z_t^b	z_b	Risk premium shock
$P_{F,t}$		Consumer's price imp. goods	Γ_t		Price adjustment costs
$P_{F,t}^f$		Importer's price imp. goods	G_t	G	Government spending
π_t	pi	Inflation in CPI	ξ_t^y	xi_y	Productivity shock
$\pi_{H,t}$	pih	Domestic inflation	$\xi_t^{C^f}$	xi_C_f	Shock to foreign dem.
$\pi_{F,t}$	pif	Imported inflation	ξ_t^r	xi_r	Monetary policy shock
$\pi_{F,t}^f$	pif_f	Foreign inflation	$\xi_t^{r^f}$	xi_rf	Mon. policy shock, foreign
B_t		Domestic bond holdings	ξ_t^b	xi_b	Shock to risk premium
B_t^f	bf	Foreign bond holdings	$\xi_t^{\pi_F^f}$	xi_pif_f	Shock to foreign inflation
S_t		Nominal exchange rate	$\xi_t^{\varepsilon_H}$	xi_vepsH	Mrkt pow. shock, prod.
Q_t	Q	Real exchange rate	$\xi_t^{\varepsilon_F}$	xi_vepsF	Mrkt pow. shock, imp.
r_t	r	Domestic interest rate	ξ_t^G	xi_G	Fiscal policy shock

Note that the code names relate to the percentage deviation from steady state in the respective variables

Table 9: Parameter descriptions

Parameter	Code	Description	Parameter	Code	Description
α	alpha	Openness	ρ_{π^f}	rho_pif_f	Pers. for. infl.
β	beta	Discount	ρ_{ε_H}	rho_vepsH	Pers. MP shock
η	eta	El. dom./for.	ρ_{ε_F}	rho_vepsF	Pers. MP shock
h	h	Habit	ρ_G	rho_G	Pers. gov. sp.
σ	sigma	Risk avers.	G	GSS	G steady s.
ε_H	vepsilon	El. goods	Q	QSS	Q steady s.
ε_F	vepsilon		P_H	phSS	P_H steady s.
φ	varphi	Inv. Frisch el.	P_F	pfSS	P_F steady s.
$\phi_{C_{H1}}$	phi_ch1	Cost adj. SS	C_F	CFSS	C_F steady s.
$\phi_{C_{H2}}$	phi_ch2	Cost adj. prev. infl	C_H	CHSS	C_H steady s.
$\phi_{C_{F1}}$	phi_cf1	Cost adj. SS	C_H^f	CH_fSS	C_H^f steady s.
$\phi_{C_{F2}}$	phi_cf2	Cost adj. prev. infl	C_H^T	CHTSS	C_H^T steady s.
ϕ	phi	Risk prem.	π^f	pi_fSS	π^f steady s.
ω_π	omega_pi	Weight infl.	r	rSS	r steady s.
ω_y	omega_y	Weight outp.	Y	ySS	Y steady s.
ω_r	omega_r	Smoothing	$\gamma_{m,H}$	phi_ch2	Marg. cost
γ_c	gammac	Imp. share cons.	$\gamma_{m,F}$	phi_cf2	Marg. cost
ρ_{r^f}	rho_rf	Pers. for. int.	$\gamma_{b,H}$	tetah1	Lag term H
ρ^Y	rho_y	Pers. prod.	$\gamma_{b,F}$	tetaf1	Lag term F
ρ^B	rho_b	Pers. risk pr.	$\gamma_{f,H}$	tetah2	Lead term H
ρ_{C^f}	rho_C_f	Pers. for. cons.	$\gamma_{f,F}$	tetaf2	Lead term F

Note that some parameters are the steady state values of the variables with the same name